Good afternoon: warm up in notebooks

Find F''(x) for each

$$F(x)=\int_e^x [t^3-3t^2+4t-\ln(t)]\,dt$$

$F'(x)=x^3-3x^2+4x-\ln x$

$F''(x)=3x^2-6x+4-\frac{1}{x}$

$$F(x)=\int_x^x \operatorname{arcsec}(3t)\,dt = 0$$

$F'(x)=0$

$F''(x)=0$

$$F(x)=\int_x^{-2} \frac{2t}{3t-4}\,dt = -\int_{-2}^{x} \frac{2t}{3t-4}\,dx$$

$$F'(x)=\frac{-2x}{3x-4} \rightarrow F''=\frac{-2(3x-4)-(-2x)(3)}{(3x-4)^2}$$

$$\frac{-6x+8+6x}{(3x-4)^2} \Leftarrow$$

$$\boxed{F''=\frac{8}{(3x-4)^2}}$$

Revisiting Net Change

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

a.) $\int_0^3 \sqrt{t+1}\,dt = \int_0^3 (t+1)^{1/2}\,dt$

$\left[\frac{2}{3}(t+1)^{3/2}\right]_0^3$

$\frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2} = \frac{14}{3}$ gal

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b) $30 + 8(3) - \frac{14}{3} = 54 - \frac{14}{3}$ gal

30: water initially there

8(3): 8 gal pumped in each hr

$\frac{14}{3}$: pumped out

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c.) $A(t) = 30 + 8t - \int_0^t \sqrt{x+1}\,dx$

d.) Find Abs max

C.N. $A'(t) = 8 - \sqrt{t+1} = 0$

$8 = \sqrt{t+1}$

$64 = t+1 \Rightarrow t = 63$

endpts

$A(0) = 30$

$63 \times 8 = 504$

$A(63) = 30 + 8(63) - \int_0^{63} \sqrt{t+1}\, dt$

$534 - 340$

≈ 200

$\left[\frac{2}{3}(t+1)^{3/2}\right]_0^{63}$

$\frac{2}{3}(64)^{3/2} - \frac{2}{3}(1)^{3/2}$

$\frac{1024}{3} - \frac{2}{3}$

$\frac{1022}{3}$

~ 340

$3\overline{)1022} = 340$ R 2

$A(120) = 30 + 8(120) - \int_0^{120} \sqrt{t+1}\, dt$

$30 + 960 - 880$

$990 - 880$

10

$-\left[\frac{2}{3}(t+1)^{3/2}\right]_0^{120}$

$-\left(\frac{2}{3}(\sqrt{121})^3 - \frac{2}{3}\right)$

$\frac{2662}{3} - \frac{2}{3}$

$\frac{2660}{3}$

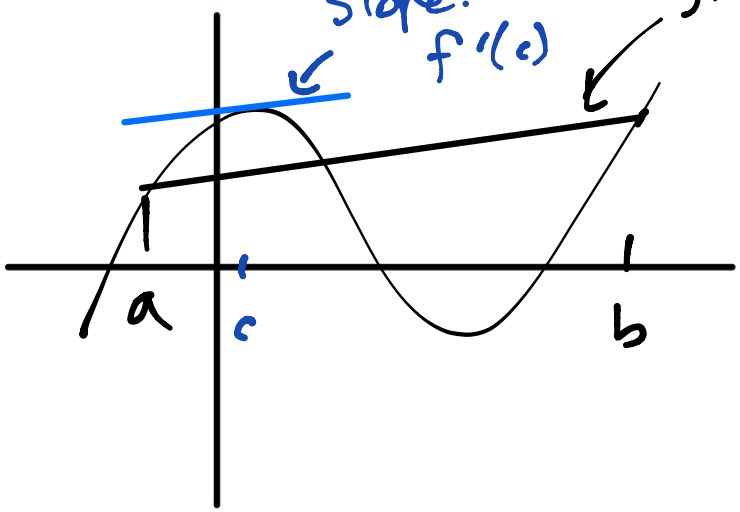
$121 \times 11 = 1331$

$3\overline{)2660} = 886\approx$

-24

26

What was the Mean Value Theorem?



Slope: $\frac{f(b)-f(a)}{b-a}$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

FTC 2

$$f'(c) = \frac{\int_a^b f'(x)\,dx}{b-a}$$

$$f'(c) = \frac{1}{b-a}\int_a^b f'(x)\,dx$$

Let $f'(x) = g(x)$

Aug. y-value that $g(x)$ has over $[a,b]$.

$$g(c) = \frac{1}{b-a}\int_a^b g(x)\,dx$$

Average Value (notes)

$\Delta x = \frac{b-a}{n}$

$n \cdot \Delta x = b-a \Rightarrow n = \frac{b-a}{\Delta x} \Rightarrow \frac{1}{n} = \frac{\Delta x}{b-a}$

How do you find the mean/average of n things?

$\bar{f} = \frac{f(x_1)+f(x_2)+\cdots+f(x_n)}{n}$

$\bar{f} = \frac{1}{n}\sum_{i=1}^{n} f(x_i)$

$\bar{f} = \frac{\Delta x}{b-a}\sum_{i=1}^{n} f(x_i)$

$\bar{f} = \frac{1}{b-a}\sum_{i=1}^{n} f(x_i)\,\Delta x$

Avg of ∞ many things?

$\lim_{n\to\infty} \bar{f} = \lim_{n\to\infty} \frac{1}{b-a}\sum_{i=1}^{n} f(x_i)\,\Delta x \rightarrow \bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$

To find the average value that f takes on over the interval [a,b],

- find $\int_a^b f(x)\,dx$ (sum of all the y-values)

- divide by $b-a$ (size of "set")

Mean Value Theorem for Integrals

If g(x) is continuous on [a,b], there exists some c in (a,b) such that

$$g(c) = \frac{1}{b - a} \int_a^b g(x)dx$$

"a continuous function must take on its average somewhere"

Find the average value of f over the interval $[4,9]$ where $f = \sqrt{x}$

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

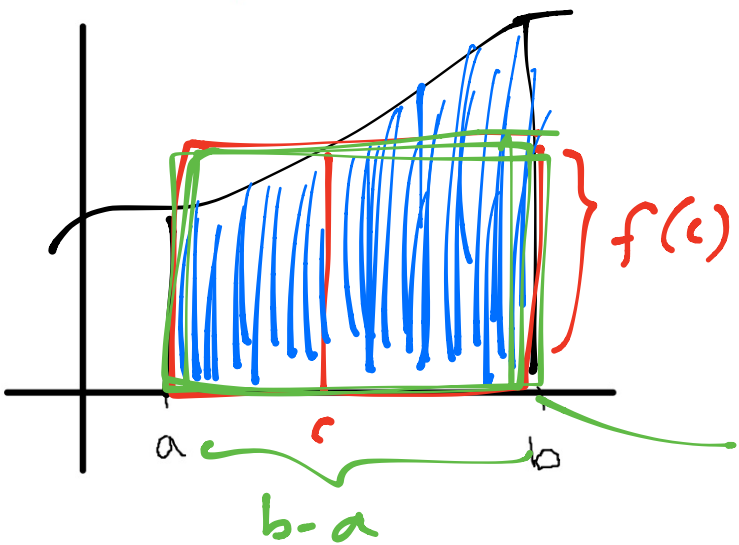
$$\frac{1}{9-4}\int_4^9 \sqrt{x}\,dx$$

$$\frac{1}{5}\left[\frac{2}{3}x^{3/2}\right]_4^9$$

$$\frac{1}{5}\left[\frac{2}{3}(\sqrt{9})^3 - \frac{2}{3}(\sqrt{4})^3\right]$$

$$\frac{1}{5}\left(18 - \frac{16}{3}\right) \rightarrow \frac{1}{5}\left(\frac{38}{3}\right) \Rightarrow \frac{38}{15}$$

Geometric Interpretation



Red "stick" is the avg height of all the blues.

Area of this Rect $= f(c) \cdot (b-a)$

$= \int_a^b f(x)\,dx$

What's on Monday's assessment??

NEW

I-A4b: Area between Curves

I-A7a: Average Value

I-A7b: Net Change Theorem

OLD

I-U7: Prop of Definite Integrals

I-U4: FTC Algebraically

I-U9: FTC Graphically

I-U3a: LRAM, RRAM

I-U3c: Riemann Sum from Table

I-A1: Basic Antiderivatives

Skills only assessed once this quarter?

I-A2b: U-Sub

I-U1: Riemann Definition

I-U2: Sum as Integral, vice versa

I-U3b: MRAM, TRAP

I-A4a Area under curve