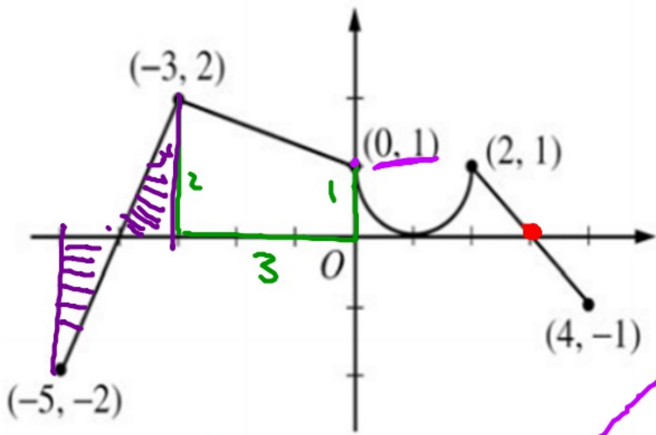
Good morning: warm up (no calculator)



Graph of f

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

$g = \int f$

$g' = f$

$g'' = f'$

(a) Find $g(0)$ and $g'(0)$. 4.5 $= 1$ $f(0) =$

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

B. $x = 3$; $g' = 0$ and g' changes $+ \to -$

$g(0) = \int_{-3}^{0} f(t)\,dt$

$x = -3, 1, 2$

g'' changes sign

c.) rel min @ $x = -4$

end points: @ $x = 4, -5$

$g(4) > 0$

$g(-5) = \int_{-3}^{-5} f(t)\,dt$

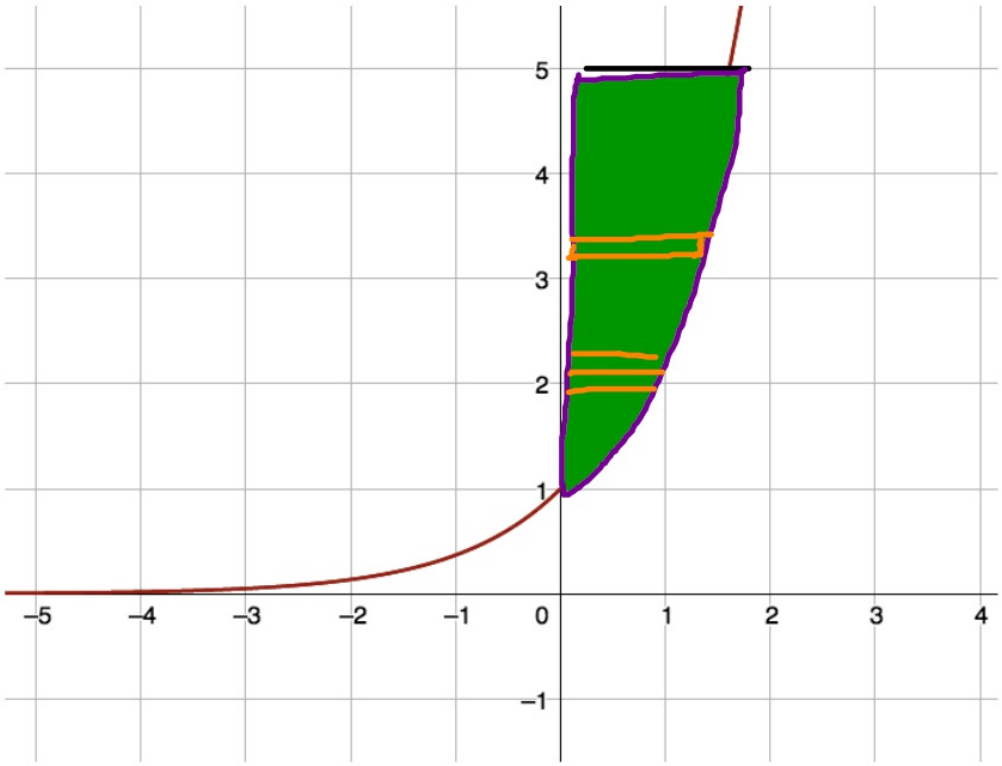
$-\int_{-5}^{-3} f(t) = 0$

$g(-4) = \int_{-3}^{-4} \Rightarrow -\boxed{\int_{-4}^{-3}} = -1$

$\boxed{\text{Abs min @ } x = -4. \quad \text{min value: } -1}$

Area, continued

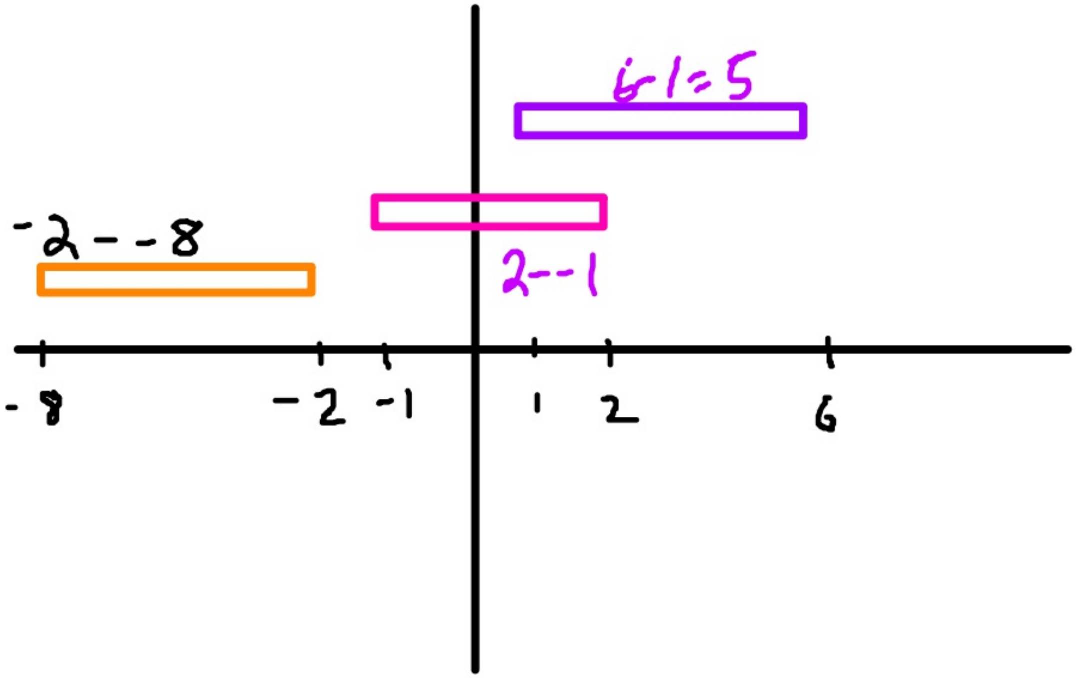
$y = e^x$



How do we find the area of the region?

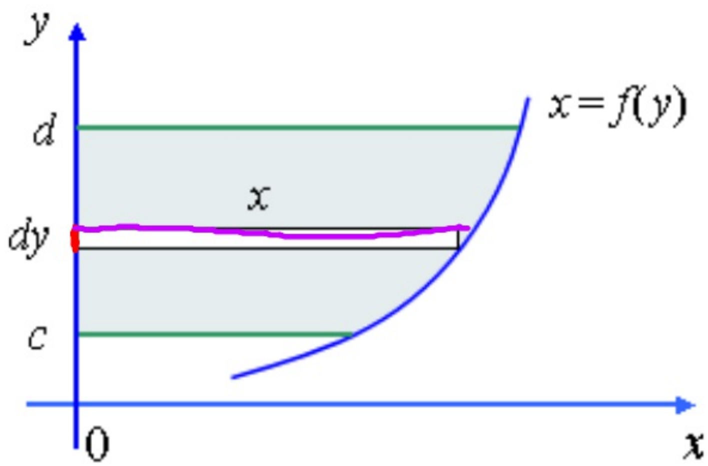
sum of horizontally oriented rectangles with width "x" and height "dy"

How 'wide' are these rectangles?



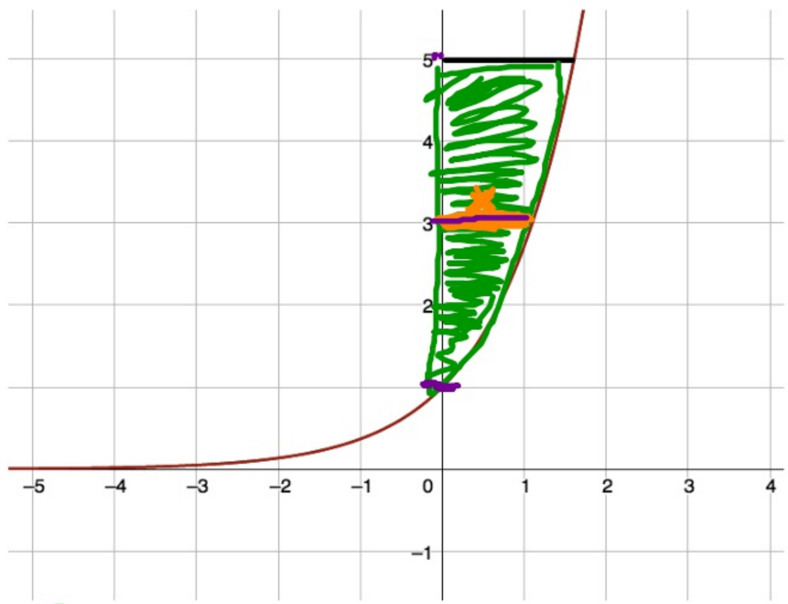
in general:
right minus left

General form



$$A = \int_c^d \overbrace{f(y)}^{\text{"}x\text{ width of rectangle}} \, dy$$

dy height



$$y = e^x$$

$$\ln y = \cancel{\ln} e^x$$

$$\underline{\ln y = x}$$

$$\int_1^5 \ln y \, dy$$

$$\boxed{4.047}$$

$$\int_0^1 2 - x^2 \, dx$$

Find the area of the region bounded by $y = \sqrt{x-1}$ and the *horizontal* lines y=1 and y=3.

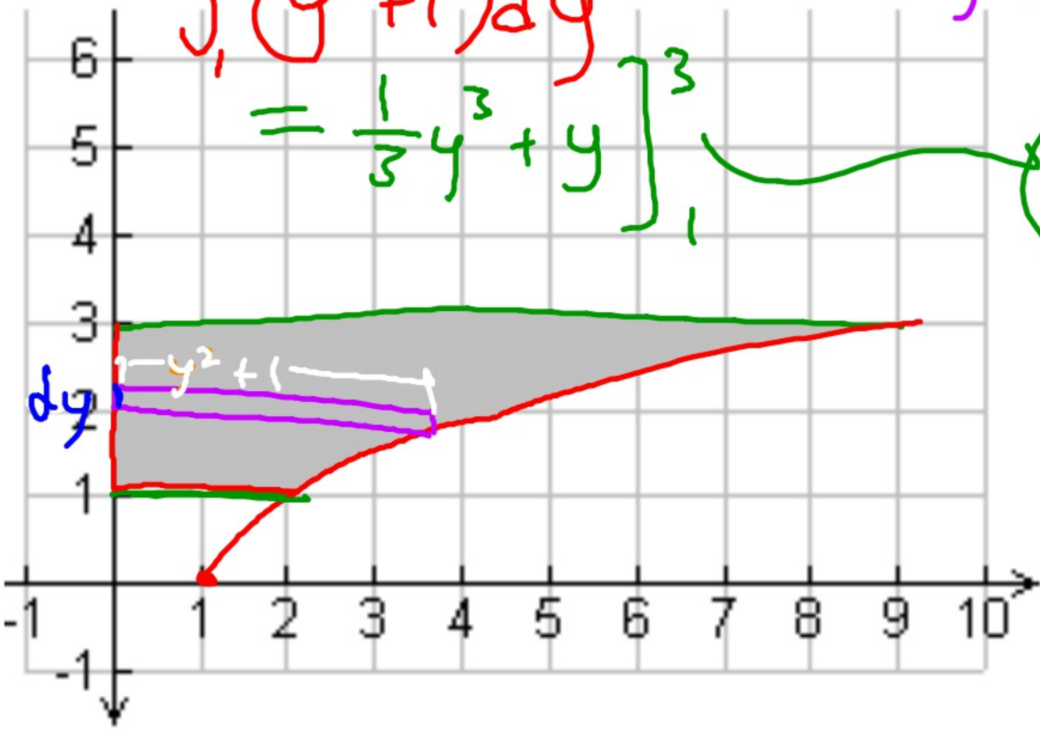
$y^2 = x-1$

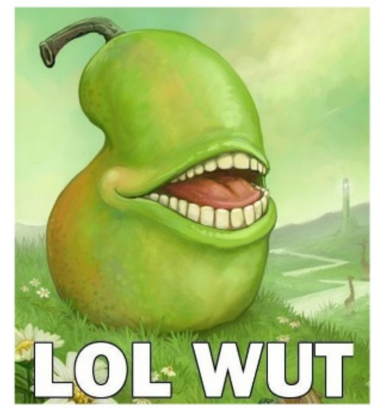
$y^2+1 = x$

$$\int_1^3 (y^2+1)\,dy$$

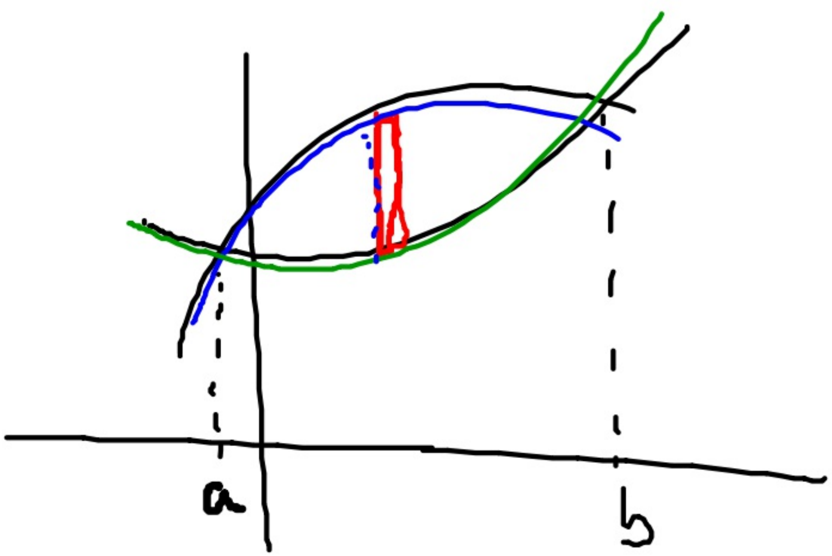
$$= \frac{1}{3}y^3 + y\Big]_1^3$$

$10\frac{2}{3}$

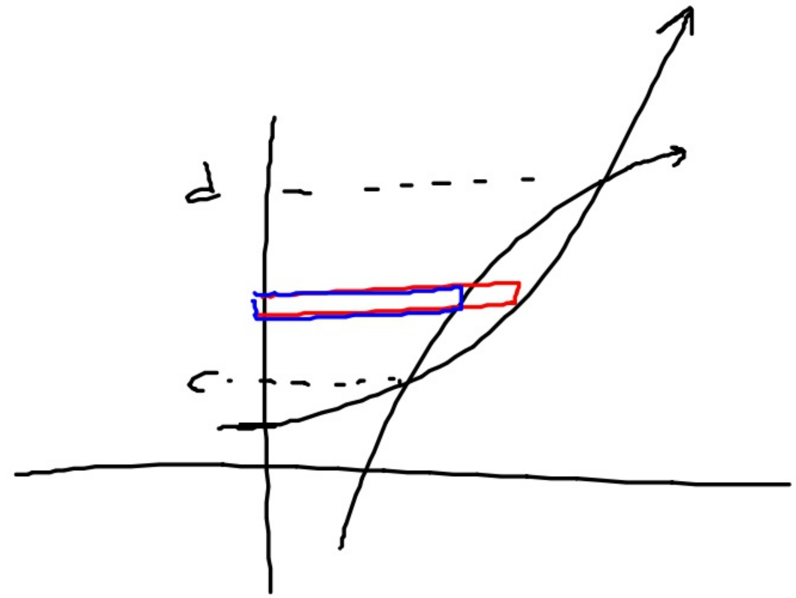




Area Between Curves



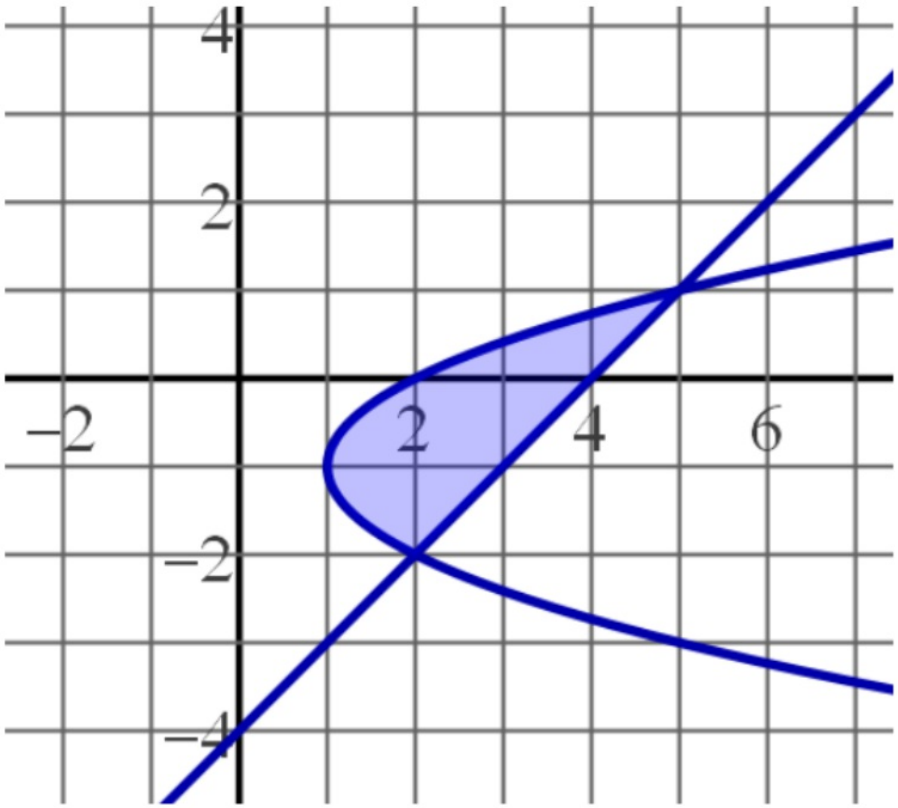
$$\int_a^b \text{top} - \text{bottom}\, dx$$



$$\int_c^d \text{right} - \text{left}\, dy$$

[redacted]

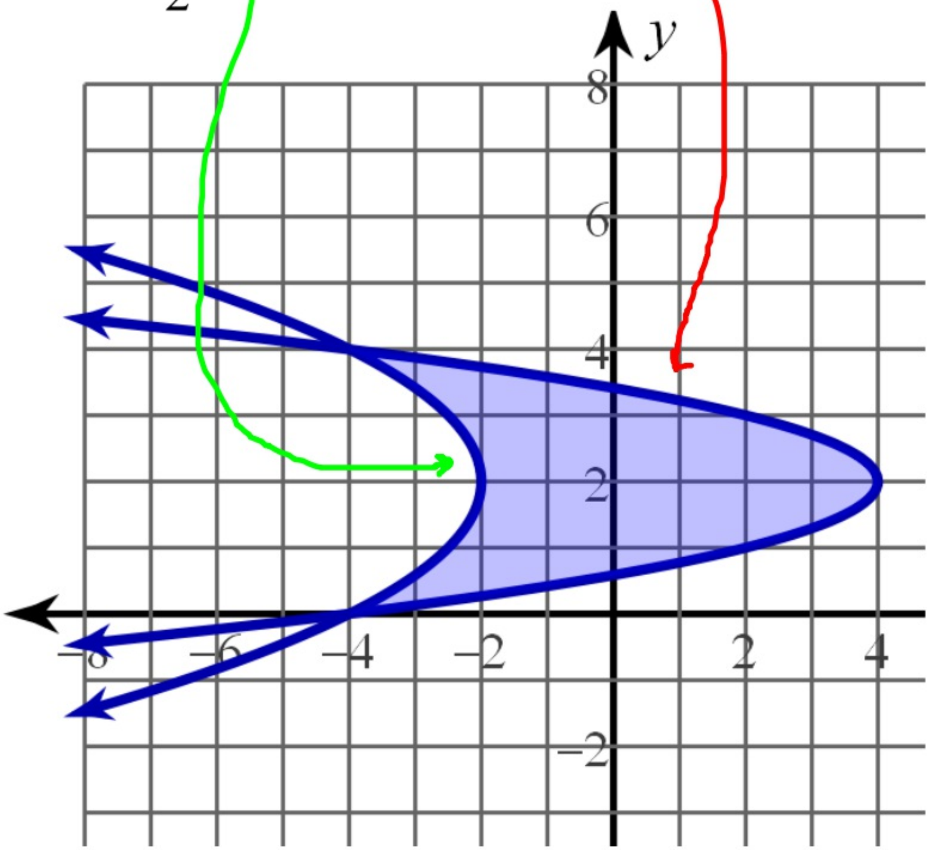
$x = y^2 + 2y + 2, \;\; x = y + 4$



<u>to do later</u>

Now try #8 on the hw handout

$$x = -\frac{y^2}{2} + 2y - 4, \quad x = -2y^2 + 8y - 4$$



<u>to do later</u>

Share with your elbow/face partner:
(+) something you have learned today
(-) something you are still not sure about

Net Change Theorem

FTC #2

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

$$+f(a) \qquad\qquad +f(a)$$

$$f(a) + \int_a^b f'(x)\,dx = f(b)$$

$$\underbrace{f(b)}_{\text{future value}} = \underbrace{f(a)}_{\text{inital value}} + \underbrace{\int_a^b f'(x)\,dx}_{\text{Sum of / accumulation of changes}}$$

Net Change Theorem

$=$

$+ \int$

Future value = Initial value + accumulated change

A honey bee population starts with 30 bees and grows at a rate of $B'(t) = \ln(12t+1)$ bees per day. How many bees are there after 1 day?

After 3 days?

$$\boxed{B(1) = ?}$$

$$B(1) = B(0) + \int_0^1 B'(t)\,dt$$

$$B(1) = 30 + \int_0^1 \ln(12t+1)\,dt$$

$$B(1) = 30 + 1.779$$

$$B(1) = 31.779 \rightarrow 31 \text{ bees}$$

$$\boxed{B(3) = ?}$$

$$B(3) = 30 + \int_0^3 \ln(12t+1)\,dt$$

$$\approx 38 \text{ bees}$$

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

a.) $R(6) = 5\sqrt{6}\cos\left(\frac{6}{5}\right) \rightarrow 4.438$ mos/day > 0

means mos. pop increasi

b.) $R'(6) = -1.913$

Concave down.

Inc. @ a decr. rate

c.) $M'(t) = R(t)$

$M(31) = M(0) + \int_0^{31} R(t)\,dt$

$1000 + -35.666$

$964.3 \rightarrow 964$

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

HW

finish as much of the AP no calculator multiple choice packet as you can by Monday