Practice Assessment Q3 #2

I-U1

1. The Riemann definition of the definite integral is given by

$$\lim_{n\to\infty} \sum_{i=0}^{n} f(x_i)\Delta x = \int_a^b f(x)dx$$

where $\Delta x = \frac{b-a}{n}$. Explain why this is so in the context of area under a curve. You may use diagrams to accompany your explanation.

Our interval has length $b-a$ along x-axis. This is split into n subintervals of Δx width, so $\frac{b-a}{n} = \Delta x$. We construct rectangles with ~~height~~ base Δx and a height (y-value) $f(x_i)$ where x_i is in each subinterval. Each rectangle thus has area $f(x_i)\cdot\Delta x$, and $\sum_{i=0}^{n} f(x_i)\Delta x$ is the sum of n such rectangles. As the limit where $n \to \infty$, this yields ∞ many rectangles with infinitely narrow bases, yielding the exact area under the curve.

I-U2

2. Write a definite integral whose value equals

$$\lim_{n\to\infty} \sum_{i=0}^{n} \sec^2\Big(\underset{a}{3} + \underbrace{\frac{5i}{n}}_{\text{varies as } i\to n}\Big)\boxed{\frac{5}{n}}_{\Delta x}$$

$\leftarrow b-a \rightarrow b-a=5$ and $a=3$

$b-3=5 \Rightarrow b=8$

$$\int_3^8 \sec^2(x)\,dx$$

3. Write the definite integral as an infinite Riemann sum using correct notation

$$\int_2^5 x^3 + \sqrt{x-1}\,dx$$

$$\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n}$$

$$\lim_{n\to\infty} \sum_{i=0}^{n} \left(2+\frac{3i}{n}\right)^3 \sqrt{2+\frac{3i}{n}-1}\cdot\frac{3}{n}$$

could simplify if desired.

$$\sqrt{1+\frac{3i}{n}}$$

$x = a + \Delta x\cdot i$

$x = 2 + \frac{3i}{n}$

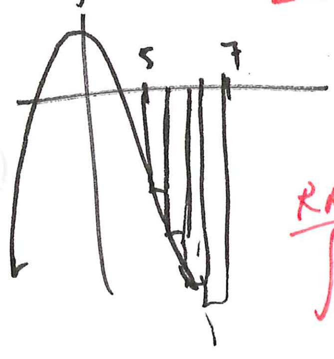
I-U3a

4. Find both the left and right rectangle approximations for $\int_5^7 \underbrace{20-(x+3)^2}_{f(x)}\,dx$ using $\overset{n}{4}$ rectangles of equal width.

$$\Delta x = \frac{7-5}{4} = \frac{1}{2}$$

LRAM / RRAM

x	5	5.5	6	6.5	7
$f(x)$	-44	-52.25	-61	-70.25	-80



$\sum f(x_i)\cdot\Delta x$

LRAM

$$\int_5^7 20-(x+3)^2dx \approx \frac{1}{2}[-44 + -52.25 + -61 + -70.25] = \frac{1}{2}[-227.5] = -113.75 \quad L$$

RRAM

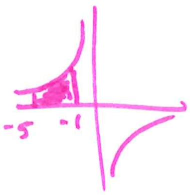
$$\int_5^7 20-(x+3)^2dx \approx \frac{1}{2}[-52.25 + -61 + -70.25 + -80] = \frac{1}{2}[-263.5] = -131.75 \quad R$$

I-U3b

5. Approximate $\int_{-5}^{-1} -\frac{3}{x}\,dx$ using 4 trapezoids of equal width.

$\Delta x = \frac{-1 - -5}{4} = \frac{4}{4} = 1$

Trap Rule $\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_0) + 2(\underbrace{\quad\ldots\quad}_{\text{all but first and last}}) + f(x_n)\right]$



$\frac{1}{2}\left[f(-5) + 2\left[f(-4) + f(-3) + f(-2)\right] + f(-1)\right]$

$\frac{1}{2}\left[\frac{-3}{-5} + 2\left[\frac{-3}{-4} + \frac{-3}{-3} + \frac{-3}{-2}\right] + \frac{-3}{-1}\right]$

$\frac{1}{2}\left[0.6 + 2(0.75 + 1 + 1.5) + 3\right] = \frac{1}{2}(10.1) = 5.05$

I-U3c

6. Selected values for $f(x)$ are given in the table below. Use a midpoint Riemann sum to approximate $\int_{10}^{90} f(x)\,dx$ using 4 intervals of equal width.

$\Delta x = \frac{90-10}{4} = \frac{80}{4} = 20$

x	10	20	30	40	50	60	70	80	90
$f(x)$	8	12	35	44	37	26	54	61	83

$\int_{10}^{90} f(x)\,dx \approx 12 \cdot 20 + 44 \cdot 20 + 26 \cdot 20 + 61 \cdot 20$

$\approx 240 + 880 + 520 + 1220$

≈ 2860

I-U5

FTC pt 2 $\int_a^b f'(x)\,dx = f(b) - f(a)$

Evaluate each definite integral.

7. $\int_{-1}^{3} x^2 - 2x\,dx$

$\left[\frac{1}{3}x^3 - x^2\right]_{-1}^{3}$

$\left[\frac{1}{3}(3)^3 - (3)^2\right] - \left[\frac{1}{3}(-1)^3 - (-1)^2\right]$

$[9 - 9] - \left[-\frac{1}{3} - 1\right] \longrightarrow 0 - \left(-\frac{4}{3}\right) = \frac{4}{3}$

8. $\int_2^5 \frac{3}{x+1}\,dx$

$3\int_2^5 \frac{1}{x+1}\,dx$

$3\left[\ln|x+1|\right]_2^5$

$3\left[[\ln|6|] - [\ln|3|]\right] \longrightarrow 3[\ln 6 - \ln 3] \xrightarrow{\text{properties of logs}} 3\ln 2$

$\ln 8 \longleftarrow \ln 2^3$

I - A2a

9.) $\int 8x \cdot \sec(3x^2)\tan(3x^2)\,dx$

want: $6x$

$\frac{8}{6}\int \frac{6}{8}\cdot 8x \; \sec(3x^2)\tan(3x^2)\,dx$

$\frac{8}{6}\int 6x \; \sec(3x^2)\tan(3x^2)\,dx$

:)

$\frac{d}{dx}\sec(x) = ?$

$\frac{8}{6}\int \sec(3x^2)\tan(3x^2)\,dx$

$\frac{4}{3}\left[\sec(3x^2) + C\right]$

$$\boxed{\frac{4}{3}\sec(3x^2) + C}$$

10.) $\int \frac{3x}{x^2+4}\,dx \Rightarrow \int 3x \cdot \frac{1}{x^2+4}\,dx \Rightarrow \frac{3}{2}\int \frac{2}{3}\,3x \cdot \frac{1}{x^2+4}\,dx$

want: $2x$

$\frac{3}{2}\int 2x \cdot \frac{1}{x^2+4}\,dx$

:)

$\frac{3}{2}\int \frac{1}{x^2+4}\,dx$

$\frac{3}{2}\left[\ln(x^2+4) + C\right]$

$$\boxed{\frac{3}{2}\ln(x^2+4) + C}$$

if you're "extra"

$\ln\left(\sqrt{x^2+4}\right)^3 + C$

why no absolute value?! hmm...