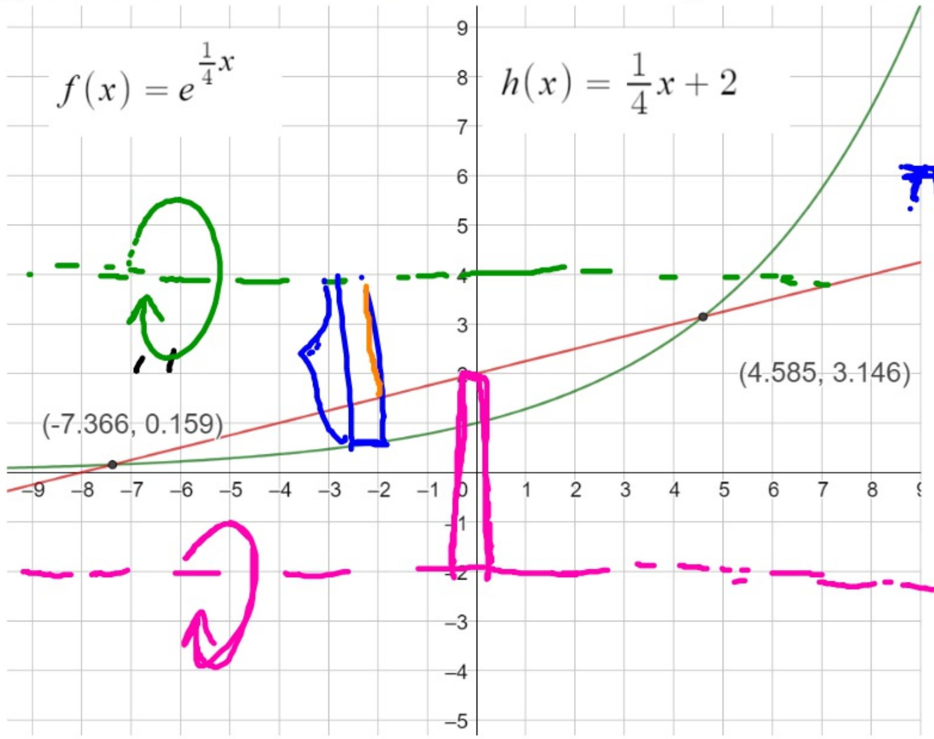
Good afternoon: warm up

Write, but do not integrate, an integral expression what will find the volume of the solid generated when S is revolved about y=4. Then repeat around y=-2.



$$\pi\int_{-7.366}^{4.585} \left(4-e^{\frac{1}{4}x}\right)^2-\left(4-\left(\frac{1}{4}x+2\right)\right)^2 dx$$

$$\pi\int_{-7.366}^{4.585} \left(\frac{1}{4}x+2--2\right)^2-\left(e^{\frac{1}{4}x}--2\right)^2 dx$$

HW answers

2010AB4

(a) Area $= \int_0^9 (6 - 2\sqrt{x})\,dx = \left(6x - \frac{4}{3}x^{3/2}\right)\Big|_{x=0}^{x=9} = 18$

(b) Volume $= \pi\int_0^9 \left((7 - 2\sqrt{x})^2 - (7-6)^2\right) dx$

2010AB1b

(a) $\int_0^2 (6 - 4\ln(3-x))\,dx = \mathbf{6.816}$ **or** $\mathbf{6.817}$

(b) $\pi\int_0^2 \left((8 - 4\ln(3-x))^2 - (8-6)^2\right) dx$

$= \mathbf{168.179}$ **or** $\mathbf{168.180}$

(d) $\pi\int_0^2 \left((6+1)^2 - (4\ln(3-x)+1)^2\right) dx$

$=217.293$

2003 AB1

Point of intersection

$e^{-3x} = \sqrt{x}$ at $(T, S) = (0.238734, 0.488604)$

(a) Area $= \int_T^1 \left(\sqrt{x} - e^{-3x}\right) dx$

$= 0.442$ or 0.443

(b) Volume $= \pi \int_T^1 \left(\left(1 - e^{-3x}\right)^2 - \left(1 - \sqrt{x}\right)^2\right) dx$

$= 0.453\pi$ or 1.423 or 1.424

2005 AB1b

The graphs of f and g intersect in the first quadrant at $(S, T) = (1.13569, 1.76446)$.

(a) Area $= \int_0^S (f(x) - g(x))\, dx$

$= \int_0^S \left(1 + \sin(2x) - e^{x/2}\right) dx$

$= 0.429$

(b) Volume $= \pi \int_0^S \left((f(x))^2 - (g(x))^2\right) dx$

$= \pi \int_0^S \left((1 + \sin(2x))^2 - \left(e^{x/2}\right)^2\right) dx$

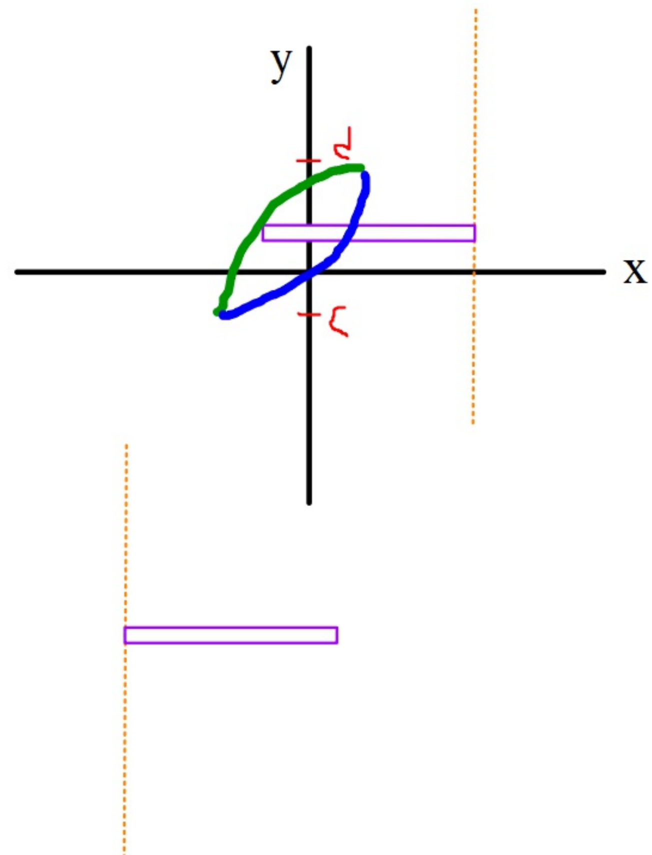
$= 4.266$ or 4.267

Washer Method for dy regions

$$\int_c^d \pi(outer\ radius)^2 - \pi(inner\ radius)^2\ dy$$

$$\pi \int_c^d (outer\ radius)^2 - (inner\ radius)^2 dy$$

outer and inner are relative, but each radius is found by RIGHT minus LEFT!



$y=e^x$

$y=\text{sqrt}(x+2)$

about $x=-3$

$(-1.961, 0.138)$
$(0.448, 1.564)$

$$\pi\int_{.138}^{1.564} (\boxed{e^x} - -3)^2 - (\boxed{\sqrt{x+2}} - -3)^2 \, dy$$

$$\pi\int_{.138}^{1.564} (\ln y + 3)^2 - (y^2 - 2 + 3)^2 \, dy$$

$y = e^x$
$\ln y = x$

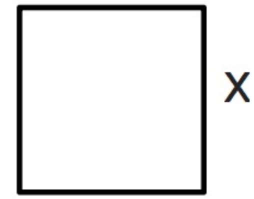
$y = \sqrt{x+2}$
$y^2 = x+2$
$y^2 - 2 = x$

Disk and Washer

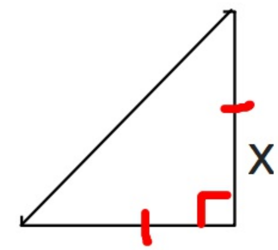
dx and dy
x-axis and y-axis
axis above region, axis below region
axis right of region, axis left of region

Review from Geometry

Find the area of a square with side x x^2



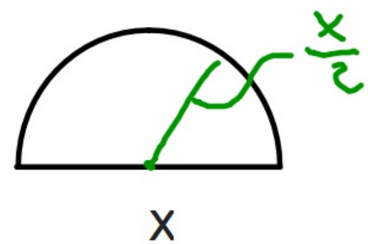
Find the area of an isosceles right triangle with leg x $\frac{1}{2}x^2$



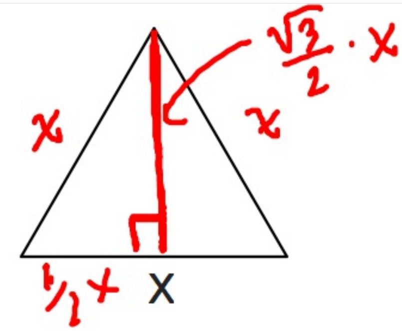
Find the area of a semicircle with diameter x

$\frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$\frac{\pi}{8}x^2$



Area of equilateral triangle with side length x



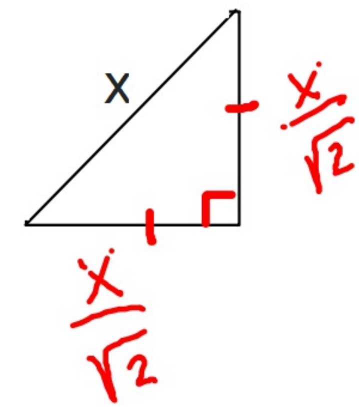
$$A = \frac{1}{2} \cdot (x) \cdot \left(\frac{\sqrt{3}}{2} \cdot x\right)$$

$$A = \frac{\sqrt{3}}{4} x^2$$

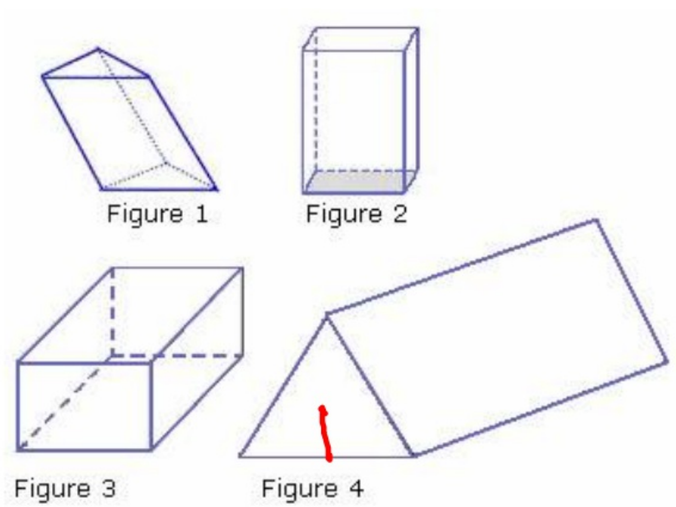
Area of isosceles right triangle with hypotenuse x

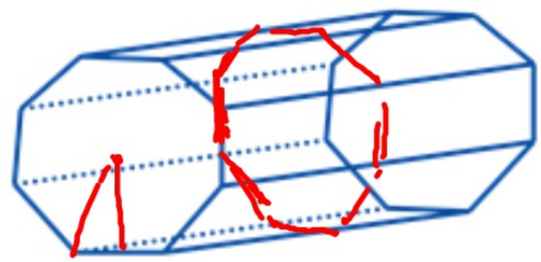
$$A = \frac{1}{2} \cdot \frac{x}{\sqrt{2}} \cdot \frac{x}{\sqrt{2}}$$

$$A = \frac{x^2}{4}$$



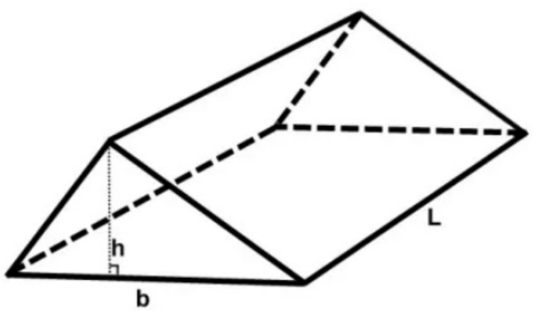
How do you find the volume of a Prism

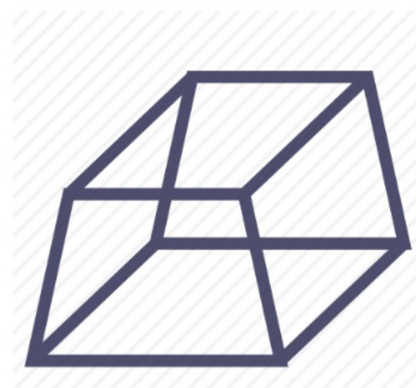




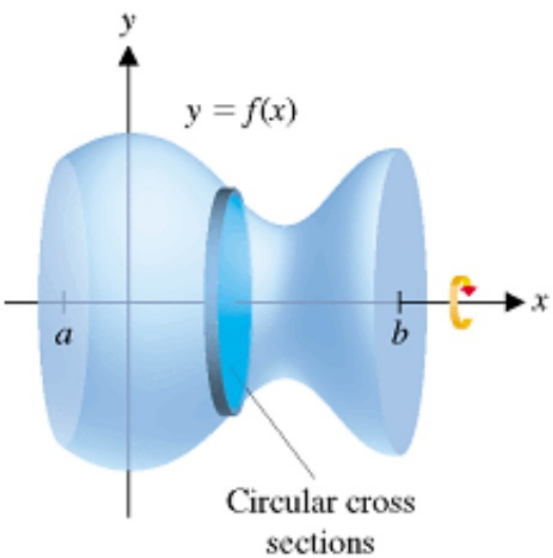
$V = B \cdot h$

B → base Area





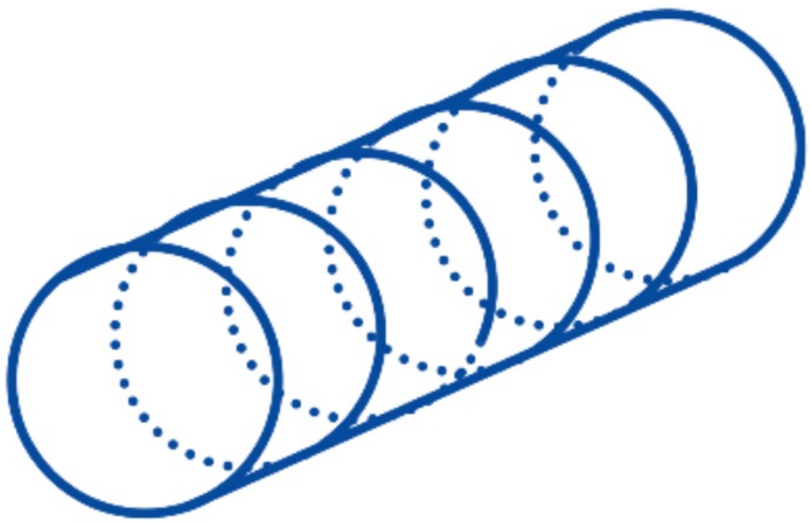
The basic premise of integrating to find volume:
Sum of disk volumes
Sum of disk area * dx (depth)



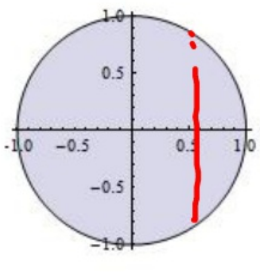
Basically just:

$$\int_a^b \pi (r)^2 \, dx$$

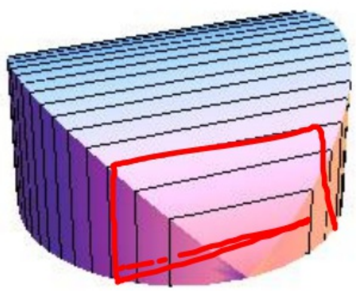
But isn't a cylinder just a circular prism?

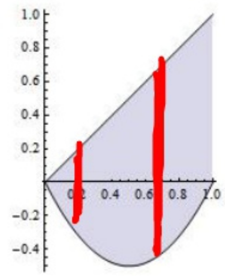


But what do shapes with non-cylindrical cross sections look like?

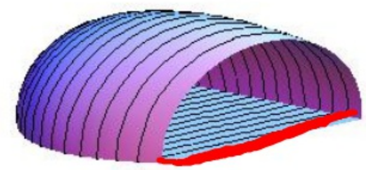


base region





base region



Two key things to remember: (1) no revolution/spinning involved
(2) the graph is flat BASE of the solid

Volume by cross sections

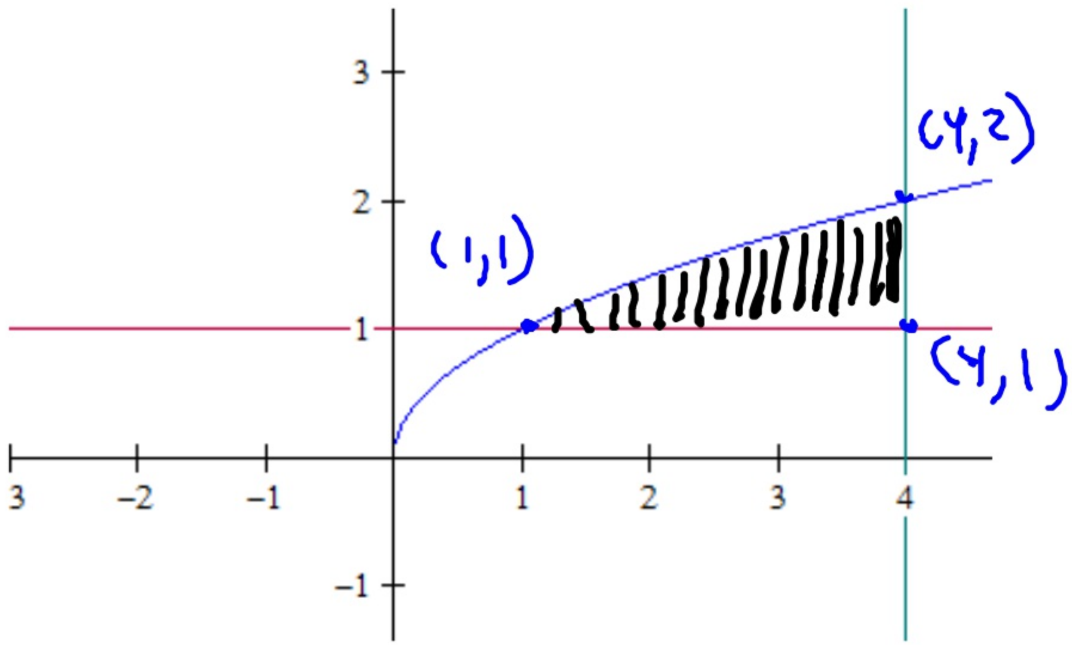
Find the face area of a cross section and integrate

$$V = \int_a^b A(x)\,dx$$

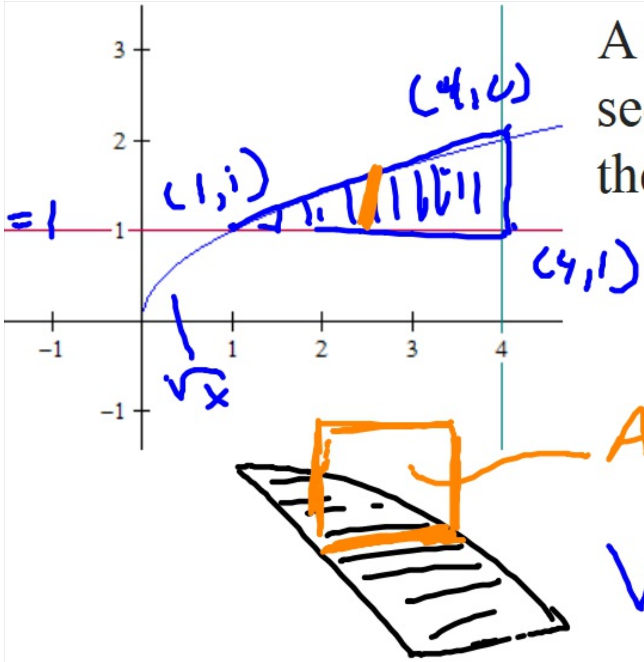
Where A(x) is the face area of a single slice

My first volume by cross sections

The region R is bound by $y = \sqrt{x}$, $y=1$, and $x=4$.



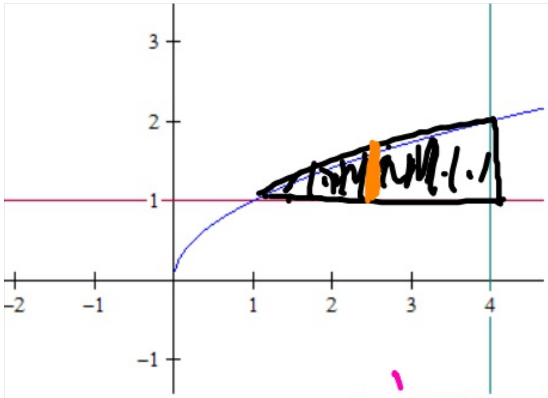
A solid with R as its base is formed where cross sections perpendicular to the x-axis are squares. Find the volume of such a solid.



top - bottom

Area? $= \left(\sqrt{x} - 1\right)^2$

$$V = \int_1^4 \left(\sqrt{x} - 1\right)^2 dx$$

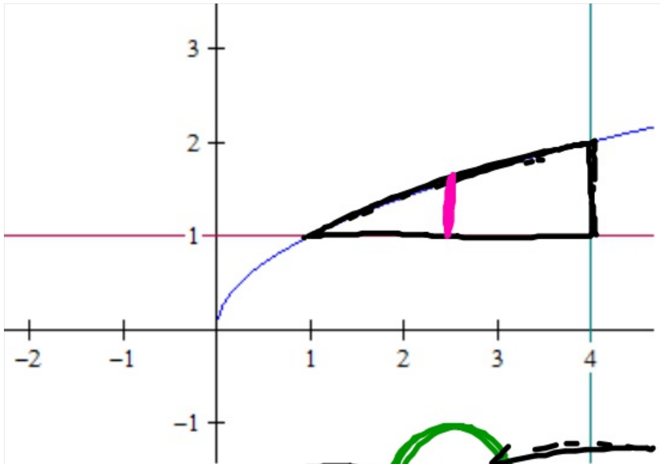


R is now the base of a solid whose cross sections perpendicular to the x-axis are...

isosceles right triangles with hypotenuse in the plane of *R*

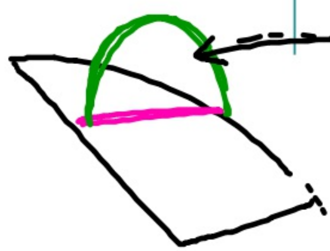
$\sqrt{x}-1$

$$\text{Area} = \frac{1}{2}(b\cdot)(h\cdot) = \int_1^4 \frac{1}{2}\left(\frac{\sqrt{x}-1}{\sqrt{2}}\right)^2 dx$$

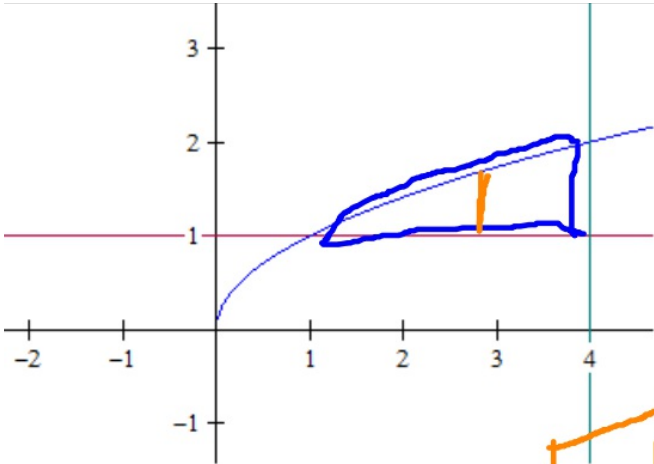


R is now the base of a solid whose cross sections perpendicular to the x-axis are...

semicircles



Area? $\int_1^4 \frac{1}{2}\pi\left(\frac{\sqrt{x}-1}{2}\right)^2 dx$



R is now the base of a solid whose cross sections perpendicular to the x-axis are...

rectangles with height twice as long as the base

Area? $= (\sqrt{x} - 1) \cdot 2(\sqrt{x} - 1)$

$A = 2(\sqrt{x} - 1)^2$

$V = \int_1^4 2(\sqrt{x} - 1)^2 \, dx$

HW

same handout as last time but do the remaining ones now

the one with the AP test questions