An object is moving in a straight line and has an acceleration modeled by
$a(t) = 2\sin(2t)$ ft/s^2

$x(3) = ?$

Find the position of the object at t=3 if the initial velocity is 4 ft/s and initial position is 2 feet.

$a(t) = 2\sin(2t)$

$\frac{dv}{dt} = 2\sin(2t)$

$\int dv = \int 2\sin(2t)\,dt$

$v = -\cos(2t) + C$

Note: first Q4 assessment scheduled for 3/28

$v(t) = - \cos(2t) + C$

Initial Vel: $4 \Rightarrow v(0) = 4 = -\cos(2\cdot 0) + C$

$4 = -1 + C \Rightarrow 5 = C$

$v(t) = -\cos(2t) + 5$

$\frac{dx}{dt} = -\cos(2t) + 5$

$dx = (-\cos(2t) + 5)\,dt$

$\int dx = \int -\cos(2t)\,dt + \int 5\,dt$

$x(t) = -\frac{1}{2}\int 2\cos(2t)\,dt + 5t + C$

$x(t) = -\frac{1}{2}\sin(2t) + 5t + 2$

"Initial pos is 2"

$x(0) = 2 = -\frac{1}{2}\cdot 0 + 5(0) + C$

$2 = C$

$x(3)$

$-\frac{1}{2}\sin(6) + 15 + 2$ ft

Visibly Random Grouping

HW

1) $\pi \int_0^2 \left(x^2\right)^2 \, dx$

$= \dfrac{32}{5}\pi \approx 20.106$

2) $\pi \int_{-2}^1 \left(-x^2 + 4\right)^2 \, dx$

$= \dfrac{153}{5}\pi \approx 96.133$

3) $\pi \int_{\frac{\pi}{2}}^{\pi} \left(2\sqrt{\sin x}\right)^2 \, dx$

$= 4\pi \approx 12.566$

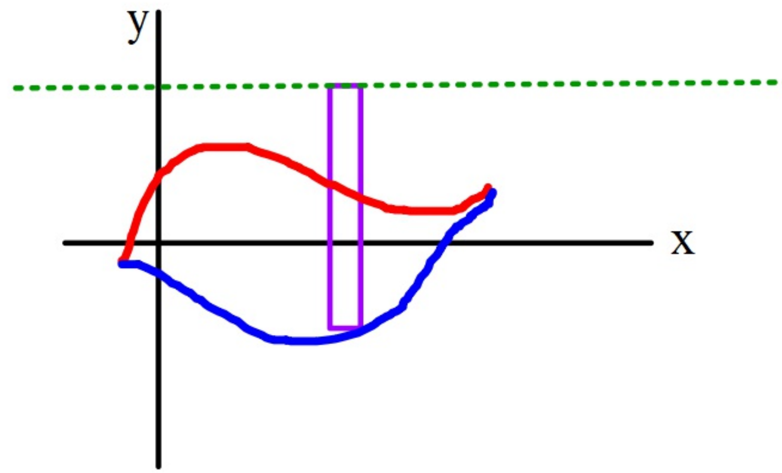
4) $\pi \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (2\csc x)^2 \, dx$

$= \dfrac{8\sqrt{3}}{3}\pi \approx 14.51$

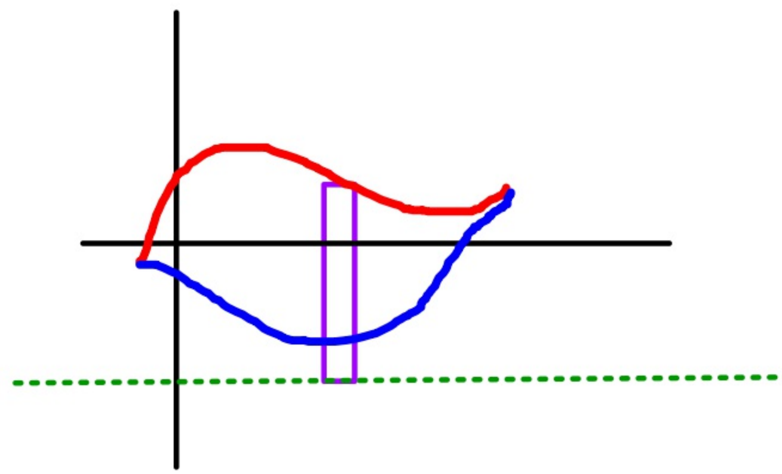
Washer method formula/idea:

$$\int_a^b \pi(outer\ radius)^2 - \pi(inner\ radius)^2\, dx$$

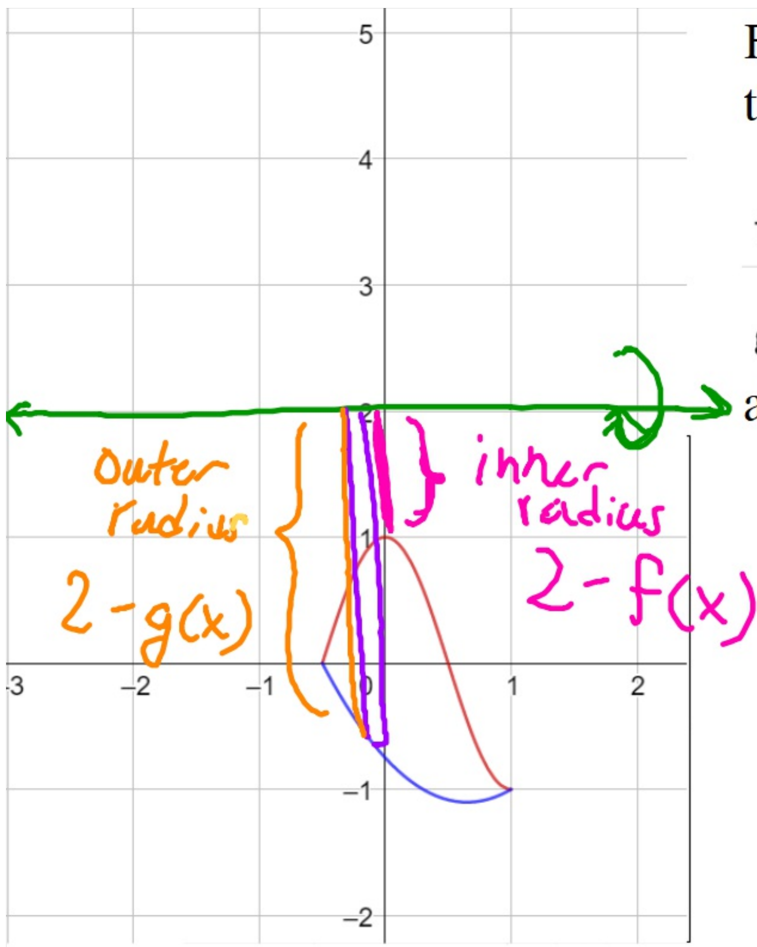
$$\pi \int_a^b (outer\ radius)^2 - (inner\ radius)^2 dx$$



outer and inner are relative, but each radius is found by top minus bottom!



1. Draw rectangle adjacent to rev. axis, identify outer vs inner radius
    1b. Draw washer if needed
2. Express outer and inner radius lengths (top minus bottom)
3. Integrate!



Find the volume of the solid generated by revolving the region bounded by

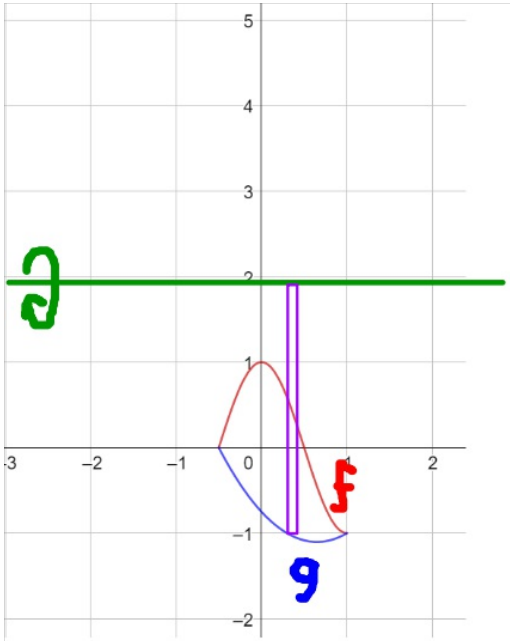
$$f(x) = \cos(\pi x)$$

$$g(x) = \frac{5}{6}x^2 - \frac{13}{12}x - \frac{3}{4}$$

about the line y=2.

$$V = \pi \int_{-1/2}^{1} \left(2 - g(x)\right)^2 - \left(2 - f(x)\right)^2 \, dx$$

$$V \approx 20.554 \; u^3$$



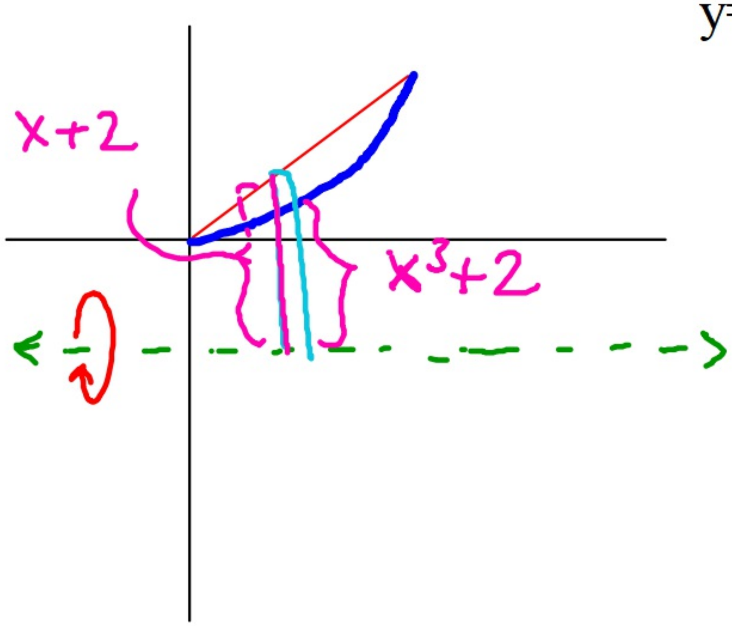
$$\pi \int_{-0.5}^{1} (2 - g(x))^2 - (2 - f(x))^2 dx$$

A simpler, fresher example

Revolve about y=-2

y=x

$y=x^3$



$$V = \pi \int_0^1 (x+2)^2 - (x^3+2)^2 \, dx$$

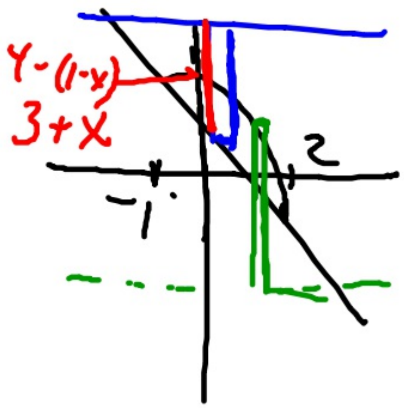
$$\pi \left[\frac{25}{21} \right]$$

More washers! Pump up the volume!

Find the volume of the solid generated by revolving the region bound by $n(x)= 1-x$ and $m(x)=3-x^2$ about

a.) the line y=4

b.) the line y=-2



a.) $\pi\int_{-1}^{2}(4-(1-x))^2-(4-(3-x^2))^2\,dx$

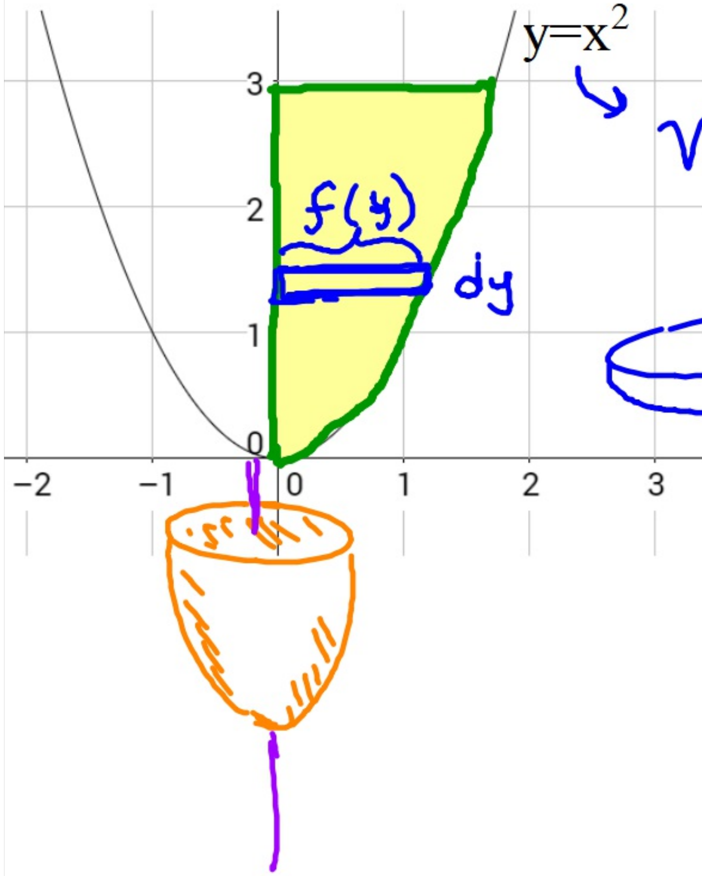
$\pi\int_{-1}^{2}(3+x)^2-(1+x^2)^2\,dx$

23.4π

b.) $V=\int_{-1}^{2}(3-x^2--2)^2-(1-x--2)^2\,dx$

$=\pi\int_{-1}^{2}(5-x^2)^2-(3-x)^2\,dx$

30.6π



Revolve the region around.... the y-axis????

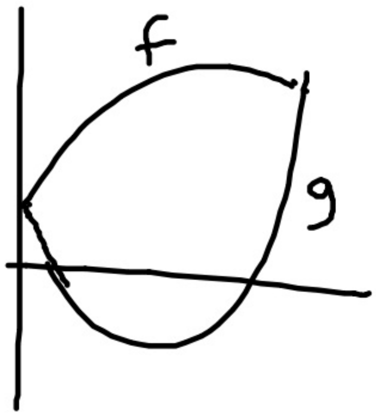
$\sqrt{y} = x$

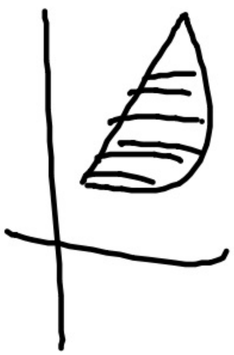
$$V = \int_0^3 \pi [f(y)]^2 \, dy$$

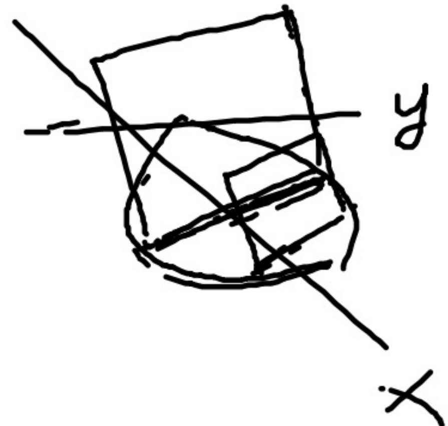
$$\pi \int_0^3 [\sqrt{y}]^2 \, dy$$

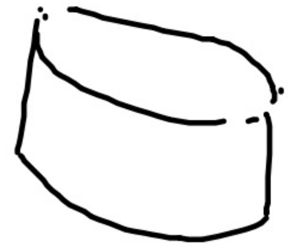
$$\pi \int_0^3 y \, dy$$

$$\pi \left[\frac{1}{2} y^2\right]_0^3 \to \pi \left[\frac{9}{2} - 0\right]$$









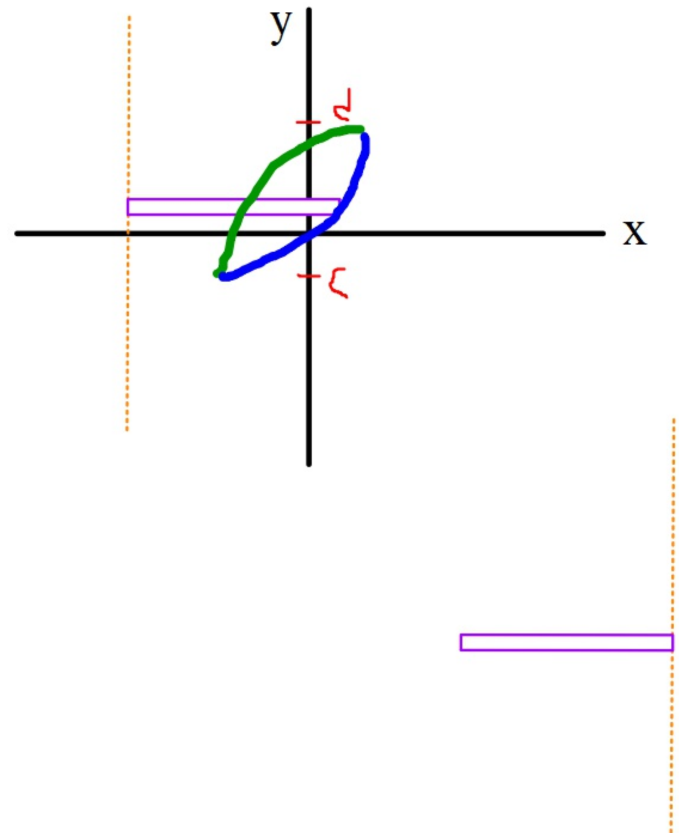
Preview of Volume by Cross Section

Washer Method for dy regions

$$\int_c^d \pi(\textit{outer radius})^2 - \pi(\textit{inner radius})^2\, dy$$

$$\pi \int_c^d (\textit{outer radius})^2 - (\textit{inner radius})^2\, dy$$

outer and inner are relative, but each radius is found by RIGHT minus LEFT!



$y=e^x$

$y=\text{sqrt}(x+2)$

about $x=-3$

HW
selected AP problems on the handout

2010AB4 no calc modded: skip c and d
2010AB1b yes calc modded: skip c
2003AB1 yes calc: skip c
2005AB1b yes calc modded: skip c and d