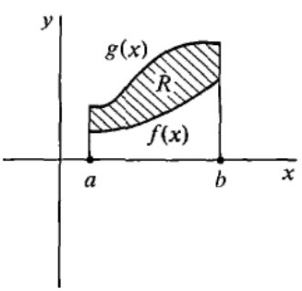
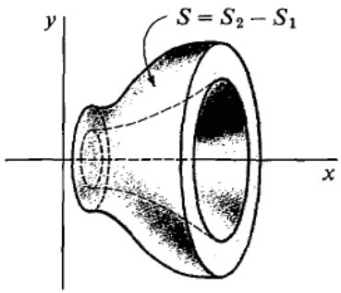
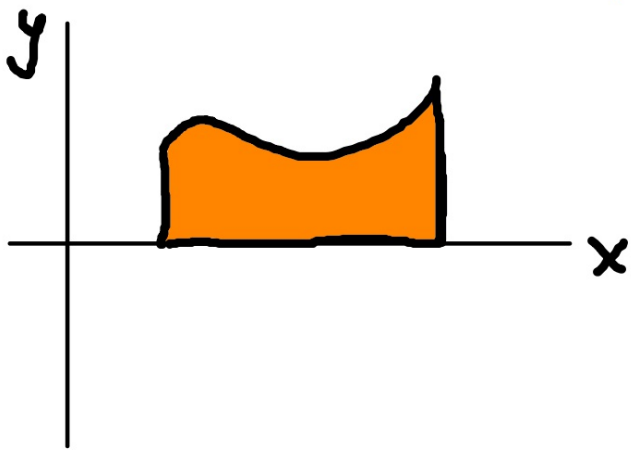
Good afternoon: no warm up, we will randomize when the bell rings and learn about how to extend the idea of area between curves to volume by revolution



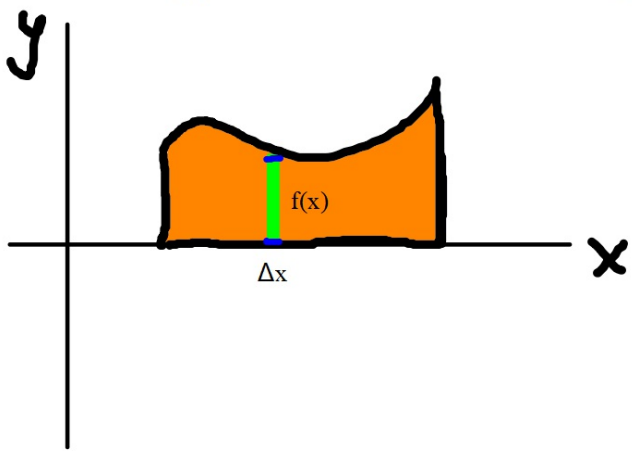


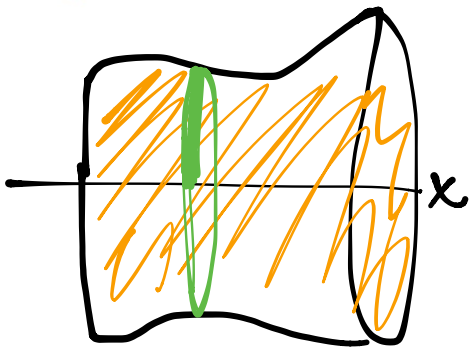
How do we find the exact area of weirdly shaped spaces?



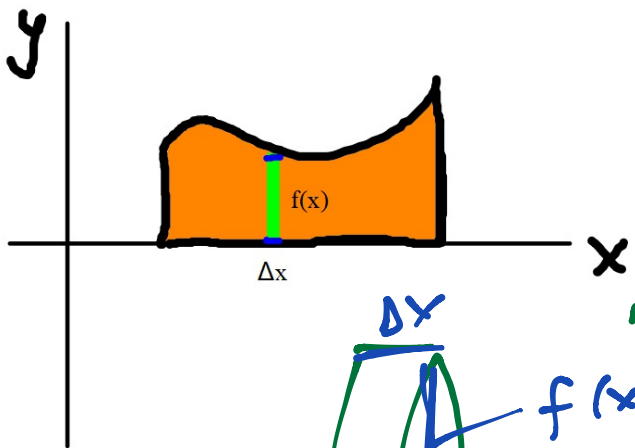
Divide into rectangles

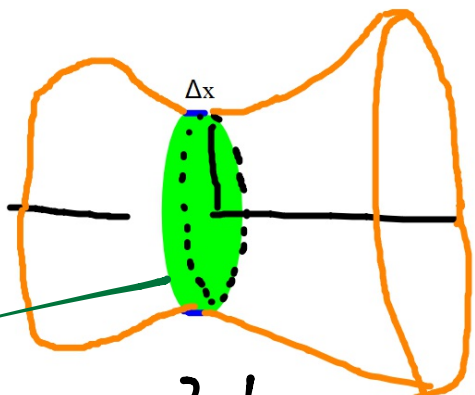
What happens when we revolve a single rectangle around the x-axis?

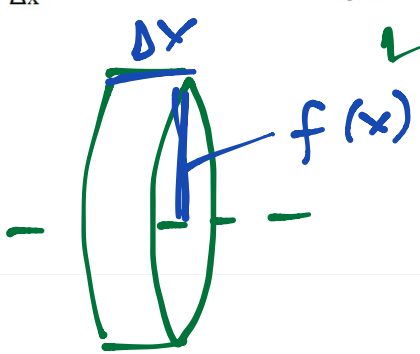




What is the volume of a single cylindrical disk?







$V = \pi r^2 \cdot h$

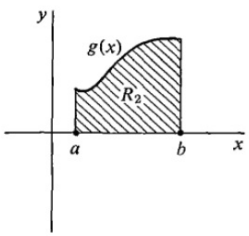
$V = \pi \cdot (f(x))^2 \cdot \Delta x$

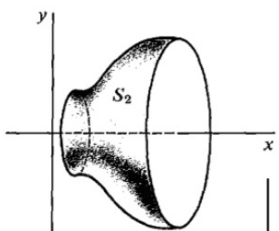
volume of single disk

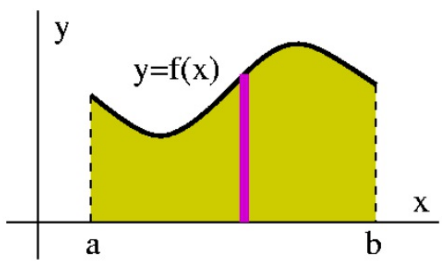
$$V = \int_a^b \pi (f(x))^2 \, dx$$

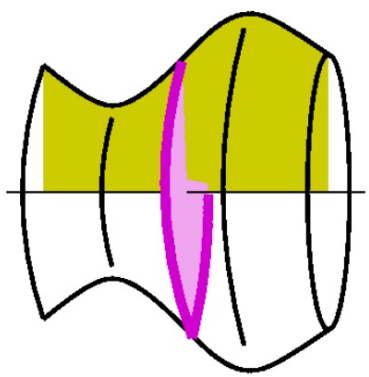
volume of all such disks

The Disk Method



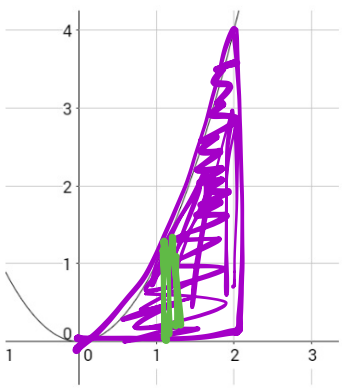


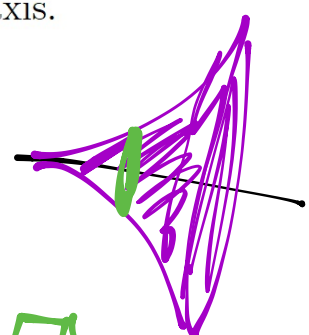


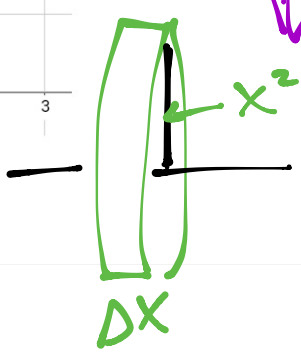



Pillsbury
Chocolate Chip COOKIES

Concrete Example: Region bounded $y = x^2$, x-axis, and $x=2$ revolved around the x-axis.







$$V = \pi (x^2)^2 \cdot dx$$

$$V = \int_0^2 \pi x^4 \cdot dx$$

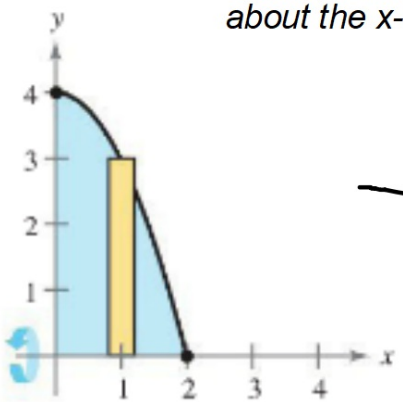
$$\pi \left[\frac{1}{5} x^5 \right]_0^2$$

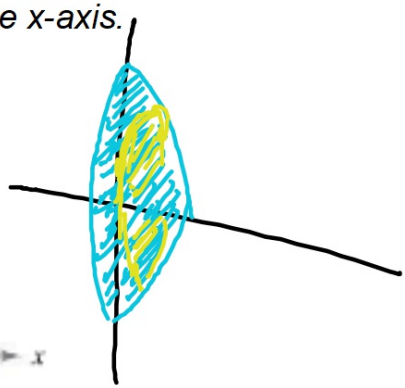
$$\pi \left[\frac{32}{5} \right] \rightarrow \frac{32\pi}{5} u^3$$

Always sketch:
- revolution axis
- single rectangle
- single disk

2. $y = 4 - x^2$

Find the volume of the solid generated by the bound region when revolved about the x-axis.





$$V = \int_0^2 \pi (4 - x^2)^2 \, dx$$

$$\pi \int_0^2 16 - 8x^2 + x^4 \, dx$$

$$\pi \left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5 \right]_0^2$$

$$\pi \left[32 - \frac{64}{3} + \frac{32}{5} \right]$$

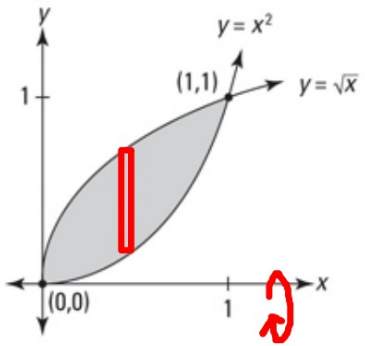
$$= \frac{256\pi}{15} \, u^3$$

Axes of Revolution: x-axis

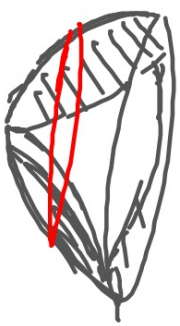
y-axis

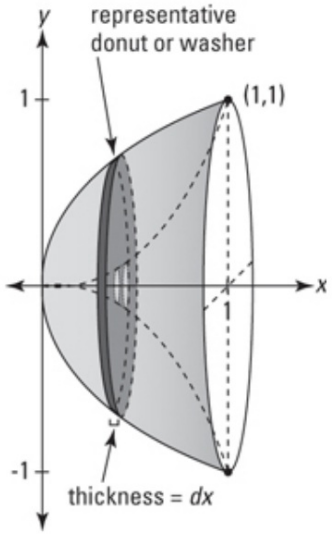
other vertical + horizontal lines

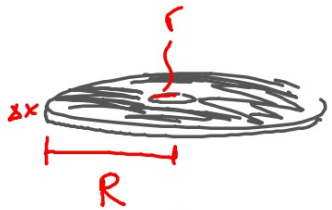
How would we
revolve this about x-axis?



now revolve this shaded area about the *x*-axis







$V = \pi R^2 \cdot \Delta x - \pi r^2 \cdot \Delta x$

$V = (\pi R^2 - \pi r^2)\Delta x$

volume of single "washer" (annulus)

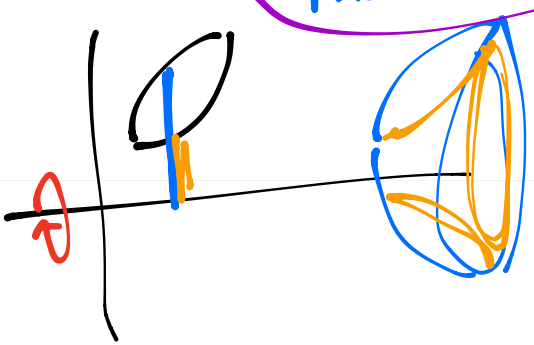
Washer Method for ~~Volume~~

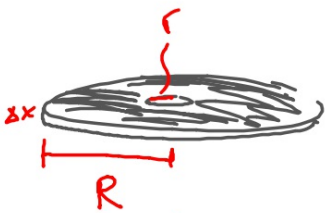
$$V = \int_a^b (\pi R^2 - \pi r^2)\,dx$$

$$\pi \int_a^b (R^2 - r^2)\,dx$$

Outer radius

inner radius

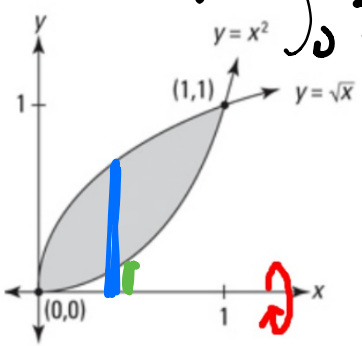




$$V = \pi R^2 \cdot \Delta x - \pi r^2 \cdot \Delta x$$

$$V = (\pi R^2 - \pi r^2)\Delta x$$

★ outer & inner are RELATIVE to Axis of REVOLUTION ★



$$V = \int_0^1 \pi(\sqrt{x})^2 - \pi(x^2)^2 \, dx$$

$$V = \pi \int_0^1 x - x^4 \, dx$$

$$\pi \left[\frac{1}{2}x^2 - \frac{1}{5}x^5 \right]_0^1$$

$$\pi \left[\frac{1}{2} - \frac{1}{5} - (0-0) \right]$$

$$\pi \left[\frac{5}{10} - \frac{2}{10} \right] \Rightarrow \boxed{\frac{3\pi}{10} u^3}$$

hw: handout #1-~~8~~ 4