

Good afternoon:
we will randomize then do this
warm up together. write it down

Find volume of solid formed by revolving

$$y = x^2 + 6x + 9$$
$$y = \sqrt{x + 3}$$

about $x = -1$

Reminders
tutoring tmrw 4-5p
assessment Weds

$$f(x) = x^2 + 6x + 9 \text{ about } x = -1$$

$$g(x) = \sqrt{x+3}$$

$$f: y = x^2 + 6x + 9$$

$$y = (x+3)^2$$

parabola, left 3

$$y = \sqrt{x+3}$$

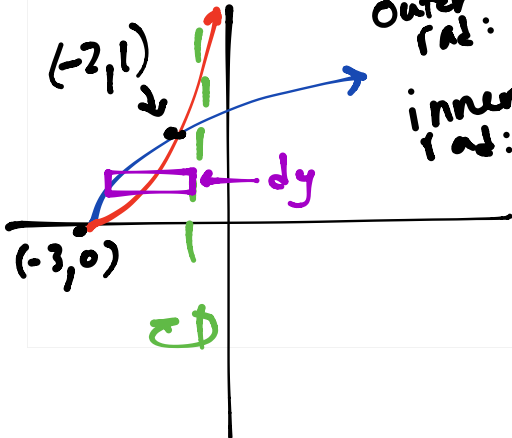
left 3

$$y = (x+3)^2$$

$$\sqrt{y} - 3 = x$$

$$y = \sqrt{x+3}$$

$$y^2 - 3 = x$$



outer rad: $-1 - f(y) \Rightarrow -1 - (\sqrt{y} - 3)$

inner rad: $-1 - g(y) \Rightarrow -1 - (y^2 - 3)$

$$V = \pi \int_0^1 (-1 - (\sqrt{y} - 3))^2 - (-1 - (y^2 - 3))^2 dy$$

$$V = \pi \int_0^1 (-1 - \sqrt{y} + 3)^2 - (-1 - y^2 + 3)^2 dy$$

$$V = \pi \int_0^1 (2 - \sqrt{y})^2 - (2 - y^2)^2 dy$$

HW

2010ab4

(c) Solving $y = 2\sqrt{x}$ for x yields $x = \frac{y^2}{4}$.

Each rectangular cross section has area $\left(3\frac{y^2}{4}\right)\left(\frac{y^2}{4}\right) = \frac{3}{16}y^4$.

$$\text{Volume} = \int_0^6 \frac{3}{16}y^4 dy$$

(d)

$$\pi \int_0^6 \left(10^2 - \left(10 - \frac{y^2}{4}\right)^2\right) dy$$

2010ab1b

(c) $\int_0^2 (6 - 4\ln(3-x))^2 dx = 26.266$ or 26.267

2003ab1

(c) Length = $\sqrt{x} - e^{-3x}$

Height = $5(\sqrt{x} - e^{-3x})$

1.554

Volume = $\int_T^1 5(\sqrt{x} - e^{-3x})^2 dx = 1.554$

2005ab1b

(c) Volume = $\int_0^S \frac{\pi}{2} \left(\frac{f(x) - g(x)}{2}\right)^2 dx$

$$= \int_0^S \frac{\pi}{2} \left(\frac{1 + \sin(2x) - e^{x/2}}{2}\right)^2 dx$$

= 0.077 or 0.078 **0.077**

(d)

$$\pi \int_1^{1.764} \left(\left(5 - \frac{1}{2} \arcsin(x-1)\right)^2 - (5 - 2 \ln x)^2 \right) dx$$

= 8.99870060173

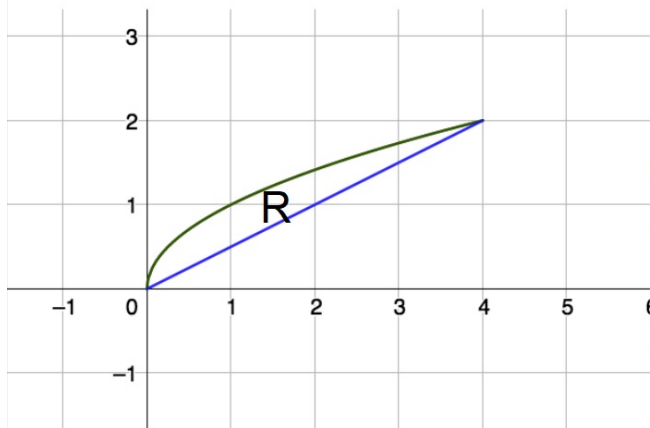
8.999

Self-Assess

How do you feel about

- disc method volume (only 1 function, or region adjacent to axis)
- disc method dy
- washer method, x -axis
- washer method, other horizontal axis
- washer method, y -axis; other vertical axis
- volume by cross section: square, rectangle, semicircle, triangle

$$y = \sqrt{x} \quad y = \frac{1}{2}x$$



Set up integral(s) to find volume if:

a.) R is revolved around x-axis

b.) R is revolved around y-axis

c.) R is revolved around $y=2$

d.) R is revolved around $x=-1$

e.) R is base, cross sections
semicircles

f.) R is base, cross sections
rectangles, 3:1 height:base

g.) R is base, cross sections
equilateral triangles

*see next page
for solutions*

$$a) \pi \int_0^4 (\sqrt{x})^2 - \left(\frac{1}{2}x\right)^2 dx \Rightarrow \pi \int_0^4 x - \frac{1}{4}x^2 dx$$

$$b) x = y^2 \quad 2y = x$$

$$\pi \int_0^2 (2y)^2 - (y^2)^2 dy = \pi \int_0^2 4y^2 - y^4 dy$$

$$c) \pi \int_0^4 \left(2 - \frac{1}{2}x\right)^2 - (2 - \sqrt{x})^2 dx$$

$$d) \pi \int_0^2 (2y - 1)^2 - (y^2 - 1)^2 dy$$

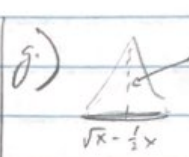
$$= \pi \int_0^2 (2y + 1)^2 - (y^2 + 1)^2 dy$$

$$e) A = \frac{1}{2} \pi R^2 = \frac{1}{2} \pi \left(\frac{\sqrt{x} - \frac{1}{2}x}{2}\right)^2 \rightarrow \int_0^4 \left(\frac{1}{2} \cdot \pi \cdot \frac{1}{4} (\sqrt{x} - \frac{1}{2}x)^2\right) dx$$

$$\frac{\pi}{8} \int_0^4 (\sqrt{x} - \frac{1}{2}x)^2 dx$$

$$f) A = 3(\sqrt{x} - \frac{1}{2}x)^2$$

$$V = \int_0^4 3(\sqrt{x} - \frac{1}{2}x)^2 dx$$



$$\frac{\sqrt{3}}{2} (\sqrt{x} - \frac{1}{2}x)$$

$$A = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} (\sqrt{x} - \frac{1}{2}x)^2$$

$$A = \frac{\sqrt{3}}{4} (\sqrt{x} - \frac{1}{2}x)^2$$

$$V = \int_0^4 \frac{\sqrt{3}}{4} (\sqrt{x} - \frac{1}{2}x)^2 dx$$

Two things to do

Need work on volume?

- Practice assessment on volume, assess is Weds.

Feel okay about volume?

- 1998 AP Mult Choice section: due after spring break