Limits

93 3. $\lim_{n\to\infty} \frac{3n^3-5n}{n^3-2n^2+1}$ is

(A) -5 (B) -2 (C) 1 (D) 3 (E) nonexistent

93 29. $\lim_{\theta\to 0} \frac{1-\cos\theta}{2\sin^2\theta}$ is

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 1 (E) nonexistent

85 37. $\lim_{x\to 0} (x\csc x)$ is

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

85 5. $\lim_{n\to\infty} \frac{4n^2}{n^2+10{,}000n}$ is

(A) 0 (B) $\frac{1}{2{,}500}$ (C) 1 (D) 4 (E) nonexistent

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73 BC 37. $\lim_{x\to 0} \frac{1-\cos^2(2x)}{x^2} =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

98 83. If $a \neq 0$, then $\lim_{x\to a} \frac{x^2-a^2}{x^4-a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

97 21. $\lim_{x \to 1} \frac{x}{\ln x}$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) nonexistent

88 BC 35. If k is a positive integer, then $\lim_{x \to +\infty} \frac{x^k}{e^x}$ is

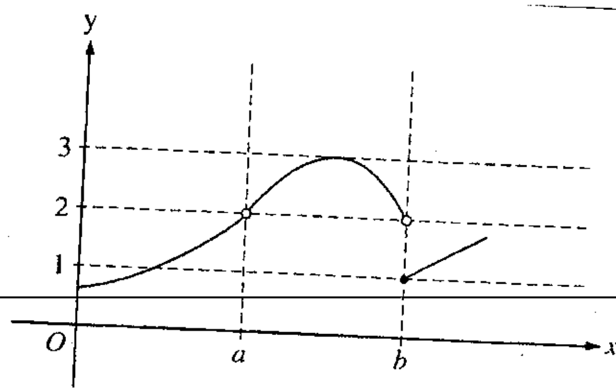
(A) 0 (B) 1 (C) e (D) $k!$ (E) nonexistent

x 85 BC 38. $\lim_{x \to \infty} \left(1+5e^x\right)^{\frac{1}{x}}$ is

(A) 0 (B) 1 (C) e (D) e^5 (E) nonexistent

93 BC 2. If $f(x)=2x^2+1$, then $\lim_{x \to 0} \frac{f(x)-f(0)}{x^2}$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent



97 15. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

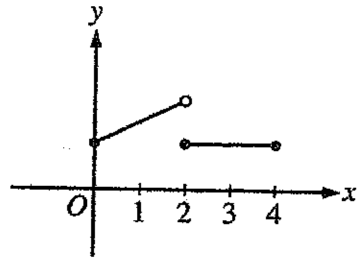
(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.



Graph of f

77. The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists.

II. $\lim_{x \to 2^+} f(x)$ exists.

III. $\lim_{x \to 2} f(x)$ exists.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

97 79. Let f be a function such that $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true?

I. f is continuous at $x = 2$.

II. f is differentiable at $x = 2$.

III. The derivative of f is continuous at $x = 2$.

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

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85 41. If $\lim_{x \to a} f(x) = L$, where L is a real number, which of the following must be true?

(A) $f'(a)$ exists.

(B) $f(x)$ is continuous at $x = a$.

(C) $f(x)$ is defined at $x = a$.

(D) $f(a) = L$

(E) None of the above

x 73 45. If f is a continuous function on $[a,b]$, which of the following is necessarily true?

(A) f' exists on (a,b).

(B) If $f(x_0)$ is a maximum of f, then $f'(x_0)=0$.

(C) $\lim_{x\to x_0} f(x) = f\left(\lim_{x\to x_0} x\right)$ for $x_0 \in (a,b)$

(D) $f'(x)=0$ for some $x\in[a,b]$

(E) The graph of f' is a straight line.

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88 41. If $\lim_{x\to 3} f(x)=7$, which of the following must be true?

x

I. f is continuous at $x=3$.
II. f is differentiable at $x=3$.
III. $f(3)=7$

(A) None (B) II only (C) III only

(D) I and III only (E) I, II, and III

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98 12. If $f(x)=\begin{cases} \ln x & \text{for } 0<x\le 2 \\ x^2 \ln 2 & \text{for } 2<x\le 4, \end{cases}$ then $\lim_{x\to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

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88 BC 5. Let f be the function defined by the following.

$$f(x)=\begin{cases} \sin x, & x<0 \\ x^2, & 0\le x<1 \\ 2-x, & 1\le x<2 \\ x-3, & x\ge 2 \end{cases}$$

For what values of x is f NOT continuous?

(A) 0 only (B) 1 only (C) 2 only (D) 0 and 2 only (E) 0, 1, and 2