Good afternoon:

warm up: find each limit. If it dne, show why.

$$f(x) = \begin{cases} 4x, & x \neq 3 \\ \cos(x), & x = 3 \end{cases}$$

$$\lim_{x \to 3} f(x)$$

$x \to 3^+$

$x \to 3^-$

$4(3^-) = \boxed{12}$

$4(3^+) =$

$$f(x) = \begin{cases} 4x, & x < 1 & = 4 \\ x^2+3, & x > 1 & = 4 \\ x+2, & x = 1 & = \cancel{3} \end{cases}$$

$$\lim_{x \to 1} f(x)$$

$\boxed{\text{limit} = 4}$

families of functions

1. constant/linear
2. absolute value
3. quadratic
4. polynomial
5. rational
6. exponential
7. logarithmic
8. trigonometric
9. inverse trigonometric

Attach to notes

(in chronological order of when
you studied them, for the most part)

Ask yourself:

"Can I do [insert calculus concept]
to each of these kinds of functions?"

- limits
- derivatives
- applications of derivatives
- antiderivatives
- definite integrals
- applications of integrals

Absolute Value Limits

$$\lim_{x \to 1} \frac{x^2 - 1}{|\boxed{x-1}|}$$

$x - 1 \geq 0$

$x \geq 1$

$$f(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & x \geq 1 \\ \dfrac{x^2-1}{-(x-1)}, & x < 1 \end{cases}$$

$$f(x) = \begin{cases} \dfrac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} \\ \dfrac{(x+1)\cancel{(x-1)}}{-\cancel{(x-1)}} \end{cases}$$

$$f(x) = \begin{cases} x+1, & x \geq 1 \\ \dfrac{x+1}{-1}, & x < 1 \end{cases}$$

Absolute values are piecewise functions in disguise!

What is the "handoff point" for an absolute value?

1. Set argument of absolute value ≥ 0, solve
2. Rewrite as piecewise function
3. Reexamine limit question

$\lim_{x \to 1} f(x)$

$\lim_{x \to 1^+} f(x) = x+1 = 1+1 = 2$

$\lim_{x \to 1^-} \dfrac{x+1}{-1} = -2$

$\neq$

dne

$$\lim_{x\to -2+} \frac{-4x-8}{|-x-2|}$$

$$-x-2 \geq 0$$
$$-x \geq 2$$
$$x \leq -2$$

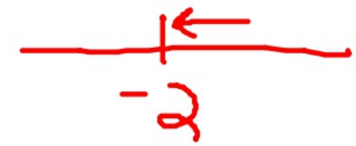
$$\begin{cases} \dfrac{-4x-8}{-x-2}, & x \leq -2 \\ \dfrac{-4x-8}{-(-x-2)}, & x > -2 \end{cases}$$

$$\begin{cases} \dfrac{4\cancel{(-x-2)}}{\cancel{-x-2}}, & x \leq -2 \\ \dfrac{4\cancel{(x-2)}}{-\cancel{(-x-2)}}, & x > -2 \end{cases}$$

$$\begin{cases} 4, & x \leq -2 \\ -4, & x > -2 \end{cases}$$

* Pick a test value to see if abs. val matters *

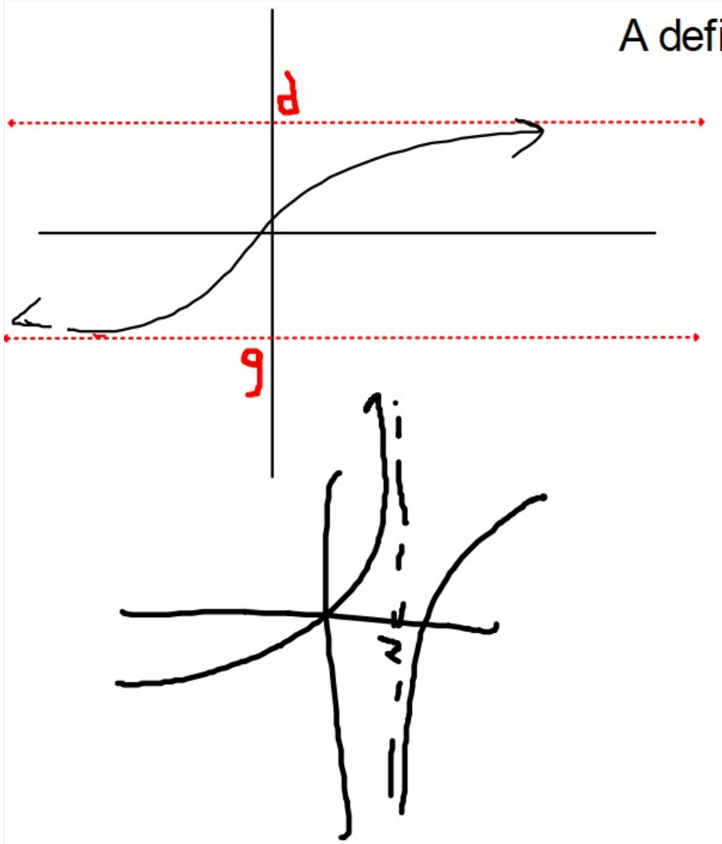
$$\lim_{x\to -2^+} = -4$$



Review:

How do you find candidates for a vertical asymptote?
How do you justify that a function has a v.a.?

set denom = 0, solve
use limits @ the candidate values



A definition of Horizontal Asymptote in terms of Limits

A function f(x) has h.a. at y=d and/or y=g if

$$\lim_{x \to \infty} f(x) = d$$

$$\lim_{x \to -\infty} f(x) = g$$

[redacted]

"Rules" for H.A. from precalc (rational functions)

Degree on top bigger:	no H.A.
Degree on bottom bigger:	H.A. is y=0
Degrees same:	H.A. is y = a/b

a and b are leading coefficients

ex:

$$y = \frac{x^5-3}{x^2-2}$$

H.A.?

$$\lim_{x\to\infty} \frac{x^5-3}{x^2-2}$$

$$\frac{\infty^5 - \cancel{3}}{\infty^2 - \cancel{2}}$$

$$= \infty$$

$$y = \frac{x^7-300}{x^9-45+x}$$

$$\lim_{x\to\infty}$$

$$\frac{\infty^7 - \cancel{300}}{\infty^9 - \cancel{45} + \cancel{\infty}}$$

$$= 0$$

$$y = \frac{3x^9-21}{7-6x^9}$$

$$\lim_{x\to\infty}$$

$$\frac{3\infty^9 - \cancel{21}}{\cancel{7} - 6\infty^9}$$

$$\frac{3\infty^9}{-6\infty^9}$$

$$-1/2$$

$$\lim_{x \to -\infty} \frac{(2x^3 \cancel{- 4x^2 + 2x + 9})}{(7x^3 \cancel{+ 2x^2 - 3x + 5})}$$

$$\frac{2(-\infty)^3}{7(-\infty)^3}$$

$$\frac{2}{7}$$

another example:

$$\lim_{x\to\infty} \frac{-x}{\sqrt{3x^2+2}}$$

$$\frac{\infty}{\sqrt{3\infty^2 + \cancel{10}}}$$

$$\frac{\infty}{\sqrt{3\infty^2}}$$

$$\frac{\cancel{\infty}}{\cancel{\infty}\sqrt{3}} \to \frac{1}{\sqrt{3}}$$

$$\frac{1}{\infty^3} - \frac{1}{\infty^2} - 10 = 0 - 0 - 10 = -1$$

$$\lim_{x\to\infty}\left(\frac{1}{x^3} - \frac{2}{x} - 10\right)$$

$$\frac{1}{x^3} - \frac{2\cdot x^2}{x\cdot x^2} - \frac{10x^3}{x^3}$$

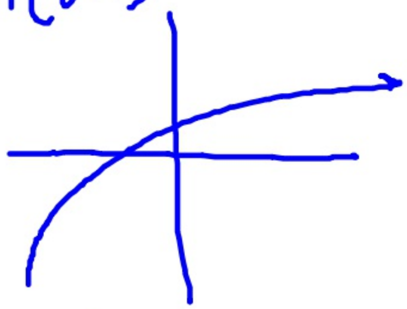
$$\frac{1 - 2x^2 - 10x^3}{x^3}$$

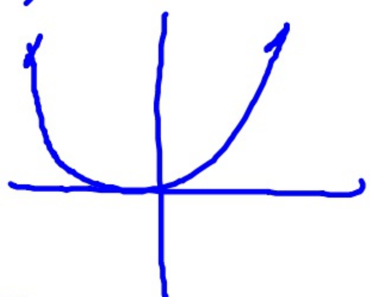
$$-x^3$$

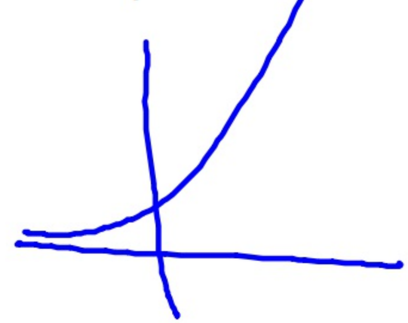
Dominance of Functions: what grows fastest?

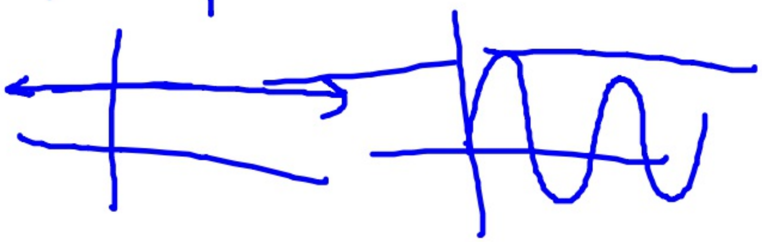
logarithmic $\ll$ polynomials $\ll$ exponentials

$\ln(x)$ $\quad$ $x^2, x^3, \ldots$









$$\lim_{x \to \infty} \frac{\sin(x)}{e^x} \qquad \frac{\pm 1}{e^{\infty}} = \frac{\pm 1}{\infty} = 0$$

$$e \approx 2.718$$

$$\lim_{x\to\infty} \frac{2^x}{3x^2} \qquad \frac{2^{\infty}}{3\infty^2} \qquad \frac{+++}{+++} = \infty$$

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