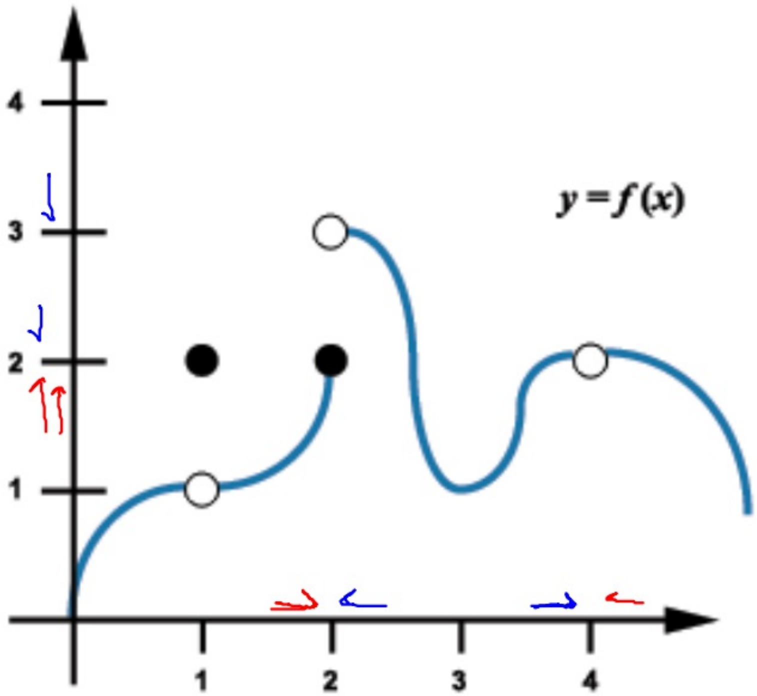
# Limits, Graphically:




$\lim_{x \to 1} f(x) = 1$

$\lim_{x \to 2} f(x)$ d.n.e.

$\lim_{x \to 2^+} f(x)$ 3

$\lim_{x \to 3} f(x)$ 1

$\lim_{x \to 4} f(x) = 2$

$f(1)$ : 2

$f(2)$ : 2

$f(4) = \varnothing$

Our limits toolkit, algebraically

- Direct substitution: always do this first
- Factor, then cancel
- Rationalization
- Numerical methods
- Special Trig limits (will learn next week


Rationalizing to find a limit (notes, continued)

$$\lim_{x\to 7} \frac{\sqrt{x+2}-3}{x-7} = \frac{\sqrt{9}-3}{7-7} = \frac{0}{0}$$

$$\sqrt{\blacksquare} \pm a \quad \text{conj} \quad \sqrt{\blacksquare} \mp a$$

$$\lim_{x\to 7} \frac{\left(\sqrt{x+2}-3\right)}{x-7} \cdot \overbrace{\frac{\left(\sqrt{x+2}+3\right)}{\sqrt{x+2}+3}}^{\text{"1"}}$$

$$\lim_{x\to 7} \frac{x+2-9}{(x-7)\left(\sqrt{x+2}+3\right)}$$

$$\lim_{x\to 7} \frac{\cancel{x-7}}{\cancel{(x-7)}\left(\sqrt{x+2}+3\right)} \Rightarrow \lim_{x\to 7} \frac{1}{\sqrt{x+2}+3} = \frac{1}{\sqrt{9}+3}$$

$$\frac{1}{3+3}$$

$$\frac{1}{6}$$

$$\lim_{x\to 4} \frac{x-\sqrt{3x+4}}{4-x} = \frac{4-\sqrt{12+4}}{4-4} = \frac{4-\sqrt{16}}{0} = \frac{0}{0} \;\; !!$$

Multiply by Conjugate

$$\lim_{x\to 4} \left(\frac{x-\sqrt{3x+4}}{4-x}\right) \cdot \left(\frac{x+\sqrt{3x+4}}{x+\sqrt{3x+4}}\right)$$

$$\lim_{x\to 4} \frac{x^2-(3x+4)}{(4-x)(x+\sqrt{3x+4})} \Rightarrow \lim_{x\to 4} \frac{x^2-3x-4}{(4-x)(x+\sqrt{3x+4})}$$

Factor quadratic

$$\lim_{x\to 4} \frac{(x-4)(x+1)}{(4-x)(x+\sqrt{3x+4})}$$

hmm...
oh! factor out a $-1$ !

$$\lim_{x\to 4} \frac{-1\cancel{(4-x)}(x+1)}{\cancel{(4-x)}(x+\sqrt{3x+4})}$$

huzzah!

$$\lim_{x\to 4} \frac{-1(x+1)}{x+\sqrt{3x+4}} = \frac{-1(4+1)}{4+\sqrt{12+4}} = \frac{-5}{4+\sqrt{16}} = \frac{-5}{8}$$

whew!!