AP Calculus: No warm up today, please check hw solutions, ask q's/correct!

1) $-\infty$ 2) $\frac{1}{3}$ 3) ∞ 4) $-\infty$

5) ∞ 6) $-\infty$ 7) $-\infty$ 8) $-\infty$

9) $-\infty$ 10) ∞ 11) $-\infty$ 12) $-\frac{1}{3}$

Reminders:
- Assessment on Monday
- DS on Weds and Thurs.

(notes)

$\infty - \infty = \infty$

$1, 2, 3, 4, 6, \ldots$

$1, 3, 5, 7, \ldots$

Indeterminate Forms

$$\frac{0}{0}$$

$$\frac{\pm\infty}{\pm\infty}$$

$$0 \cdot (\pm\infty)$$

$$-\infty + \infty \text{ or } \infty - \infty$$

$$0^0; 1^{\pm\infty}; \infty^0$$

Contrast with these:

$$\frac{1}{0^{\pm}} = \pm\infty; \quad \frac{1}{\pm\infty} = 0$$

$$\lim_{x \to a} \frac{x - a}{x^4 - a^4}$$

$$\lim_{x \to a} \frac{x-a}{(x^2)^2 - (a^2)^2} \Rightarrow \lim_{x \to a} \frac{x-a}{(x^2+a^2)(x^2-a^2)}$$

$$\lim_{x \to a} \frac{x-a}{(x^2+a^2)(x+a)(x-a)}$$

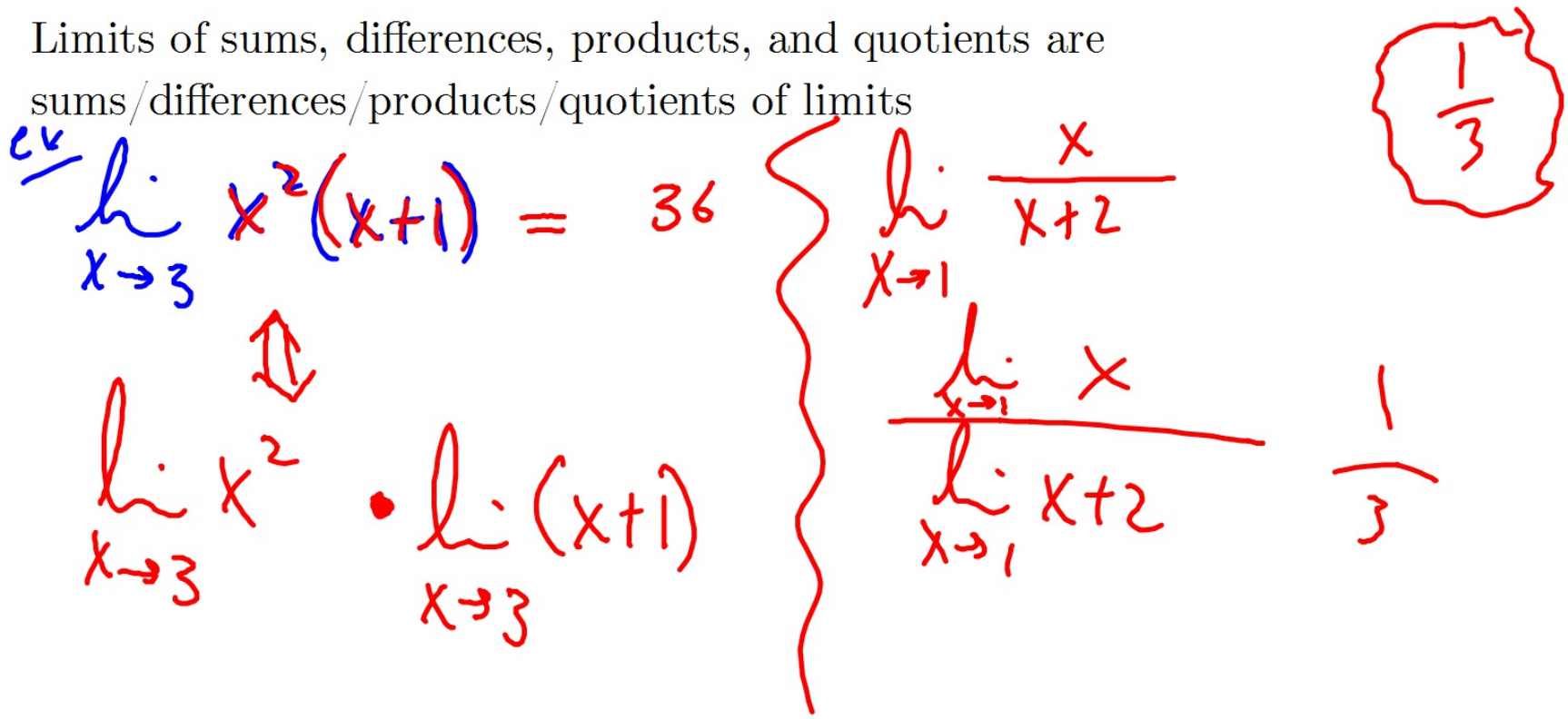
$$\lim_{x \to a} \frac{1}{(x^2+a^2)(x+a)} = \frac{1}{(a^2+a^2)(2a)} = \frac{1}{4a^3}$$

$2a^2$

Properties of Limits

Use common sense!

Limits of sums, differences, products, and quotients are sums/differences/products/quotients of limits



Two Special Trig Limits!!! (the squeeze theorem)

Graph the following functions in your calculator

Y1= 1

Y2 = sin(x)/x $\quad \dfrac{\sin(x)}{x}$

Y3= $\cos^2(x)$

Around the origin, which function is always the greatest?

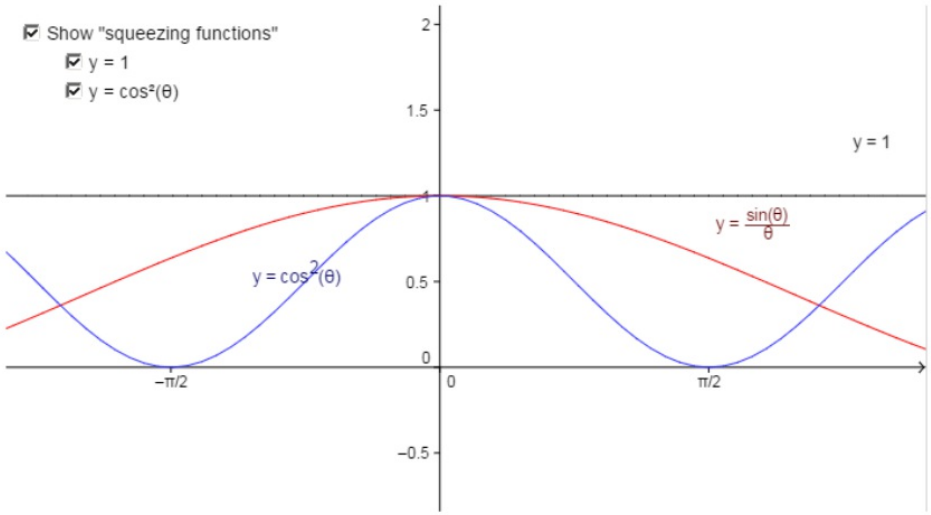
Around the origin, which function is always the least?

Around the origin, which function is always in the middle?

What is Y1(0)?

What is Y3(0)?

What is Y2(0)?



$$\lim_{x\to 0}\cos^2(x) \le \lim_{x\to 0}\frac{\sin(x)}{x} \le \lim_{x\to 0} 1$$

$$1 \le \lim_{x\to 0}\frac{\sin x}{x} \le 1$$

$$\Rightarrow \lim_{x\to 0}\frac{\sin x}{x} = 1$$

MEMORIZE THESE!

get them tattooed

$$\lim_{x\to 0}\frac{\sin(x)}{x} = 1$$

$$\lim_{x\to 0}\frac{1-\cos(x)}{x} = 0$$

Example:

$\sin(ax) \neq a \cdot \sin(x)$

$$\lim_{x\to 0} \frac{\sin(3x)}{x} \cdot \frac{3}{3}$$

$$\lim_{x\to 0} \frac{3 \cdot \sin 3x}{3x} \Rightarrow \lim_{x\to 0} 3 \cdot \frac{\sin 3x}{3x}$$

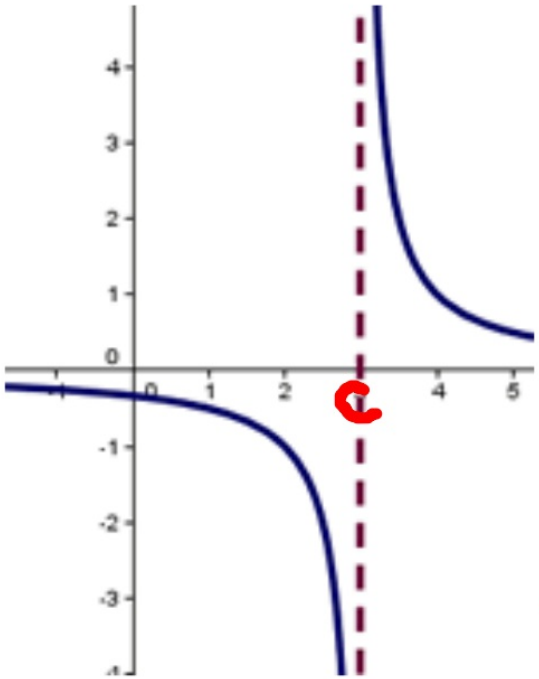
$$\lim_{x\to 0} 3 \cdot \lim_{x\to 0} \frac{\sin 3x}{3x}$$

by tattoo, $= 1$

$$3 \cdot 1 = 3$$

$$\lim_{x \to 0} x \csc x$$

$$\lim_{x \to 0} \frac{\tan 2x}{x}$$

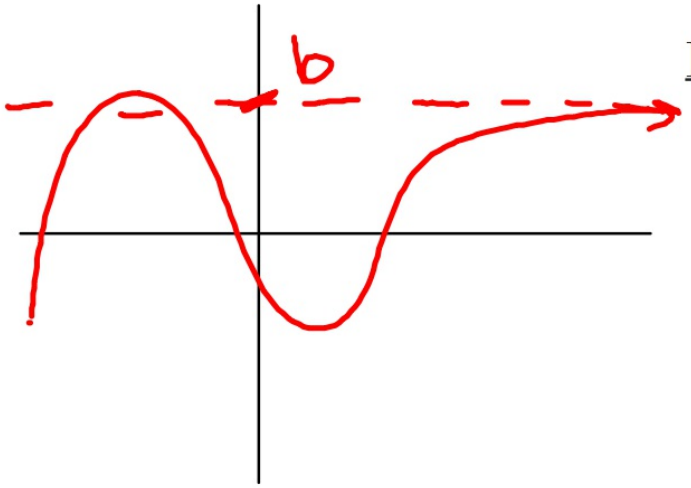


Definition of Vertical Asymptote

$x = c$ is a v.a. iff

any of the following is true:

- $\lim_{x \to c^+} f(x) = \pm\infty$
- $\lim_{x \to c^-} f(x) = \pm\infty$



Definition of Horizontal Asymptotes

$y = b$ is h.a. iff

$$\lim_{x \to \infty} f(x) = b.$$

or

$$\lim_{x \to -\infty} f(x) = b.$$

H.A. "Rules" from Pre-cal

Degree on top bigger:	no H.A.
Degree on bottom bigger:	H.A. is y=0
Degrees same:	H.A. is $y = a/b$

where a and b are leading coefficients

$$\lim_{x \to -\infty} \frac{(2x^3 - 4x^2 + 2x + 9)}{(7x^3 + 2x^2 - 3x + 5)}$$

What will test questions on this look like?

Find any vertical asymptote(s). **Justify Your Answer** using limits

$f(x) = (x+5)/(x^2+4x-5)$

Find any horizontal asymptote(s). **Justify Your Answer** using limits

$f(x) = (x+5)/(x^2+4x-5)$