NO CALCULATORS

The student should be able to...

...write a piecewise function from a rational function that includes an absolute value; then find its one-sided limits

See pg 4

...evaluate limits (finite and infinite) and discuss continuity from a "generic graph" (See "Limits: A Graphical Approach" worksheet)

See website for new example problem.

...evaluate limits (both existant and one-sided) of all the function families

Tips:

- be able to mentally "picture" the graph made by each type)
- "massage the limit" to reveal cancellations (removable discontinuities)
  two tips for massages: factor the denominator; also rational the *numerator*
- Use properties of limits: p57 table

See pg 5

- ...justifiably find asymptotes of functions using the limit definitions.

See below

- ...demonstrate Continuity at a point using the three conditions necessary

- ....explain, with evidence, why a given function has a zero (that is, f(c) = 0, for some c in (a,b)) for a continuous function

See pg 9

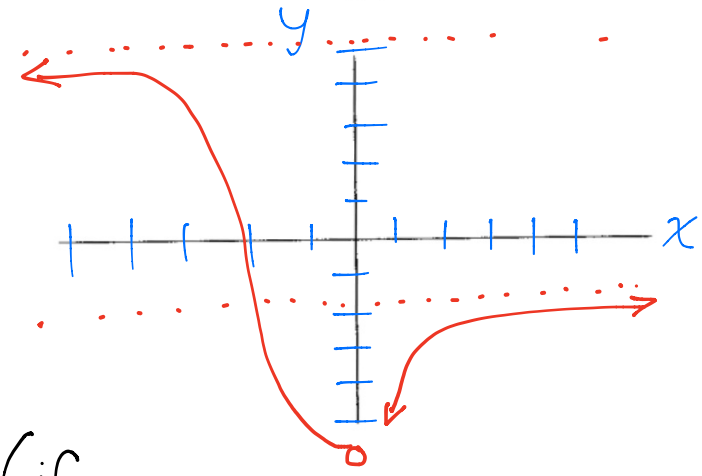
...given either a function (perhaps piecewise) or a graph of a function, identify with justification the types of discontinuities present. (Jump, removable, infinite)

see below

...sketch a possible graph for a function given certain requirements (expressed as limits)

ex: Graph a possible graph for the function $P(x)$ satisfying the following conditions:

$$\lim_{x\to\infty} P(x) = -2 \quad \lim_{x\to-\infty} P(x) = 5 \quad \lim_{x\to 0^+} P(x) = -\infty \quad \lim_{x\to 0^-} P(x) = -6$$



Classify (if Discontinuous)

$$f(x) = \begin{cases} x & , x \le -2 \\ -x^2 - 2x & x > -2 \end{cases}$$

$f(-2) = -2$

$\lim_{x\to -2^-} f \neq \lim_{x\to -2^+} f$

So, Jump Discontinuity.

$\rightarrow \lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} x = -2$

$\rightarrow \lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} -x^2 - 2x = -4 + 4 = 0$

...solve a system of equations to find a and b to make a given piecewise function continuous over the entire real line:

ex: Find a and b to make f(x) everywhere continuous.

$$f(x) = \begin{cases} 5bx - 6a, \text{ if } x < -2 \\ -3b - 4ax, \text{ if } x = -2 \\ 5x - 1, \text{ if } x > -2 \end{cases}$$

Continuity @ $x=-2$

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x) = f(-2)$$

$$\lim_{x\to -2^-} 5bx - 6a = \lim_{x\to -2^+} 5x-1 = -3b - 4a(-2)$$

$$-10b - 6a = -11 = -3b + 8a$$

$$\begin{cases} (-10b - 6a = -11)\,4 \\ (-3b + 8a = -11)\,3 \end{cases}$$

$$+\begin{cases} -40b - 24a = -44 \\ -9b + 24a = -33 \end{cases}$$

$$\frac{-\cancel{49}b}{\cancel{-49}} = \frac{-77}{-49}$$

$$b = 11/7$$

$$-3\left(\frac{11}{7}\right) + 8a = -11$$

$$\frac{-33}{7} + 8a = \frac{-77}{7}$$

$$\frac{1}{8}\left(8a = \frac{-44}{7}\right)\frac{1}{8}$$

$$a = \frac{-44}{56}$$

$$a = -11/14$$

$$\begin{array}{r} 14 \\ 4\overline{)56} \\ -4\downarrow \\ 16 \end{array}$$

Rewrite as a P-wise and find $\lim_{x \to -4^-} g(x)$

$$g(x) = \frac{3|x+4|}{-x-4}$$

When is the absolute value positive?

$x+4>0$

$x>-4$

Now we know our p-wise conditions

$$g(x) = \begin{cases} \text{abs. val is neg.} & x<-4 \\ \text{abs. val is positive} & x>-4 \end{cases}$$

$$= \begin{cases} \frac{(3)(-1)(x+4)}{-x-4}, & x<-4 \\ \frac{3(x+4)}{-x-4}, & x>-4 \end{cases}$$

factor -1 out to make cancellation more clear

$$\begin{cases} \frac{3(-x-4)}{-x-4}, & x<-4 \\ \frac{3(-1)(-x-4)}{-x-4}, & x>-4 \end{cases}$$

$$g(x) = \begin{cases} 3, & x<-4 \\ -3, & x>-4 \end{cases}$$

$$\lim_{x \to -4^-} g(x)$$

$$\lim_{x \to -4^-} 3 = \boxed{3}$$

#1

$$\lim_{x \to 3^-} \frac{3}{x^2-6x+9} \to = \lim_{x \to 3^-} \frac{3}{(x-3)(x-3)} = \frac{3}{(3^- - 3)(3^- - 3)}$$

$$= \frac{3}{(0^-)(0^-)}$$

$$= \frac{3}{0^+} = \infty$$

#2

$$\lim_{x \to 0} 3x + 2 + x^{-2}$$

$$\to \lim_{x \to 0} 3x + 2 + \frac{1}{x^2}$$

$$3(0) + 2 + \frac{1}{0^2}$$

$$2 + \frac{1}{0^+} = \infty$$

#3

$$\lim_{x \to 0} \frac{\sqrt{x+9} - 3}{x} = \frac{0}{0}$$

Rationalize the num.

$$\frac{\sqrt{x+9} - 3}{x} \cdot \frac{\sqrt{x+9} + 3}{\sqrt{x+9} + 3}$$

(Continued...)

$$\frac{x+9-9}{x\left(\sqrt{x+9}+3\right)}$$

$$\frac{\cancel{x}}{\cancel{x}\left(\sqrt{x+9}+3\right)} \rightarrow \frac{1}{\sqrt{x+9}+3}$$

$$\lim_{x\to 0} \frac{1}{\sqrt{x+9}+3} = \frac{1}{\sqrt{0+9}+3}$$

$$= \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{3+3}$$

$$= 1/6$$

given: $\lim_{x \to c^-} f(x) = -2/3$

$\lim_{x \to c^+} f(x) = -2/3$

$\lim_{x \to c} g(x) = 1/5$

a.) find $\lim_{x \to c} \frac{f(x)}{g(x)}$

b.) find $\lim_{x \to c} f(x) \cdot g(x)$

a.) $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{-2/3}{1/5}$

$= \frac{-2}{3} \cdot \frac{5}{1}$

$= -10/3$

b) $\lim_{x \to c} f(x) \cdot g(x) = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$

$$-\frac{2}{3} \cdot \frac{1}{5} = -2/15$$

$$\lim_{x \to \infty} \frac{3x^3 - 9x^2 + 5x}{12x^3 - 9x^2 + 5x^7}$$

$$\lim_{x \to \infty} \frac{3x^3 - 9x^2 + 5x}{5x^7 + 12x^3 - 9x^2}$$

$$\frac{\infty^3}{\infty^7} \Rightarrow 0$$

$$\lim_{x \to -3/4^-} \frac{-5 + 4x}{3 + 4x} = \frac{-5 + 4(-3/4^-)}{3 + 4(-3/4^-)} = \frac{-5 - 3^-}{3 + -3^-} = \frac{-8}{3 - 3^-} = \frac{-8}{0^+}$$

$$= -\infty$$

J

Demonstrate why

$f(x) = 3x^2 - 2x - 6$

has a root in the interval $[1, 2]$.

<u>I.V.T.</u>

- f is continuous b/c it's a polynomial.
- $f(2) = 12 - 4 - 6 = 2$
- $f(1) = 3 - 2 - 6 = 1 - 6 = -5$
- $f(2) \neq f(1)$

So I.V.T. says, there exists some c in $[1,2]$ such that $f(c) = 0$ because 0 is in the interval $[f(1), f(2)]$ or $[-5, 2]$.

A root is where $f(x) = 0$, so c is a root.