1.17 Have You Reached the Limit?

304. Estimate the value of $\lim_{x\to\infty}(\sqrt{x^2+x+1}-x)$ by graphing or by making a table of values.

305. Estimate the value of $\lim_{x\to\infty}(\sqrt{x^2+x}-\sqrt{x^2-x})$ by graphing or by making a table of values.

306. Consider the function $f(x)=\begin{cases} x^2-1 & -1\le x<0 \\ 2x & 0<x<1 \\ 1 & x=1 \\ -2x+4 & 1<x<2 \\ 0 & 2<x<3. \end{cases}$

a) Graph this function.

b) Does $f(-1)$ exist?

c) Does $\lim_{x\to -1^+} f(x)$ exist?

d) Does $\lim_{x\to -1^+} f(x)=f(-1)$?

e) Is f continuous at $x=-1$?

f) Does $f(1)$ exist?

g) Does $\lim_{x\to 1^+} f(x)$ exist?

h) Does $\lim_{x\to 1^+} f(x)=f(1)$?

i) Is f continuous at $x=1$?

j) Is f defined at $x=2$?

k) Is f continuous at $x=2$?

l) At what values of x is f continuous?

m) What value should be assigned to $f(2)$ to make the function continuous at $x=2$?

n) To what new value of $f(1)$ be changed to remove the discontinuity?

307. Is $F(x)=\dfrac{|x^2-4|x}{x+2}$ continuous everywhere? Why or why not?

308. Is $F(x)=\dfrac{|x^2+4x|(x+2)}{x+4}$ continuous everywhere? Why or why not?

FIND THE CONSTANTS a AND b SUCH THAT THE FUNCTION IS CONTINUOUS EVERYWHERE.

309. $f(x)=\begin{cases} x^3 & x\le 2 \\ ax^2 & x>2 \end{cases}$

310. $g(x)=\begin{cases} \dfrac{4\sin x}{x} & x<0 \\ a-2x & x\ge 0 \end{cases}$

311. $f(x)=\begin{cases} 2 & x\le -1 \\ ax+b & -1<x<3 \\ -2 & x\ge 3 \end{cases}$

312. $g(x)=\begin{cases} \dfrac{x^2-a^2}{x-a} & x\ne a \\ 8 & x=a \end{cases}$

1.18 Multiple Choice Questions on Limits

313. $\lim_{x\to\infty} \dfrac{3x^4 - 2x + 1}{7x - 8x^5 - 1} =$

A) ∞ B) $-\infty$ C) 0 D) $\frac{3}{7}$ E) $-\frac{3}{8}$

314. $\lim_{x\to 0^-} \dfrac{1}{x} =$

A) ∞ B) $-\infty$ C) 0 D) 1 E) does not exist

315. $\lim_{x\to 1/3} \dfrac{9x^2 - 1}{3x - 1} =$

A) ∞ B) $-\infty$ C) 0 D) 2 E) 3

316. $\lim_{x\to 0} \dfrac{x^3 - 8}{x^2 - 4} =$

A) 4 B) 0 C) 1 D) 3 E) 2

317. In order for the line $y = a$ to be a horizontal asymptote of $h(x)$, which of the following must be true?

A) $\lim_{x\to a^+} h(x) = \infty$

B) $\lim_{x\to a^-} h(x) = -\infty$

C) $\lim_{x\to\infty} h(x) = \infty$

D) $\lim_{x\to -\infty} h(x) = a$

E) $\lim_{x\to -\infty} h(x) = \infty$

318. The function $G(x) = \begin{cases} x - 3 & x > 2 \\ -5 & x = 2 \\ 3x - 7 & x < 2 \end{cases}$ is not continuous at $x = 2$ because

A) $G(2)$ is not defined

B) $\lim_{x\to 2} G(x)$ does not exist

C) $\lim_{x\to 2} G(x) \neq G(2)$

D) $G(2) \neq -5$

E) All of the above

319. $\lim_{x\to 0} \dfrac{3x^2 + 2x}{2x + 1} =$

A) ∞ B) $-\infty$ C) 0 D) 1 E) $\frac{3}{2}$