Reverse Chain Rule for Antiderivatives

Review

$\int 5x^4\,dx$

$\boxed{x^5 + C}$

$\cancel{5}\frac{1}{\cancel{5}}x^5$

x^5

$\int 7x^3\,dx$

$7 \cdot \frac{1}{4}x^4$

$\boxed{\frac{7}{4}x^4 + C}$

$\int (x+1)^2\,dx$

$(x+1)(x+1)$

$\int (x^2+2x+1)(dx)$

$\frac{1x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} + x + C$

$\frac{1}{3}x^3 + x^2 + x + C$

$\int (x+1)^2\,dx$

$\frac{1}{3}(x+1)^3 + C$

$\frac{1}{\cancel{3}}\cancel{3}(x+1)^2 \cdot 1$

$(x+1)^2$

ex/

Reverse Chain Rule

$$\int (x+1)^{99}\,dx$$

$$\boxed{\frac{1}{100}(x+1)^{100} + C}$$

$\downarrow d/dx$

$$\frac{1}{\cancel{100}}\cancel{100}(x+1)^{99}$$

$$(x+1)^{99}$$

$$\int (2x+5)^{99}\,dx$$

$$\frac{1}{100}(2x+5)^{100}\cdot\frac{1}{2}$$

$$\frac{1}{200}(2x+5)^{100} + C$$

$$\frac{1}{200}\,100(2x+5)^{99}\cdot 2$$

$$\frac{1}{\cancel{2}}(2x+5)^{99}\cdot\cancel{2}$$

ex/

$$\frac{1}{3}\int 3\cdot 2x^5(x^6+1)^3\,dx$$

$6x^5$

$$\frac{1}{3}\int 6x^5(x^6+1)^3\,dx$$

$$\frac{1}{3}\cdot\frac{1}{4}(x^6+1)^4$$

$$\frac{1}{12}(x^6+1)^4 + C$$

$$\int x^3\,dx$$

$$\frac{1}{4}x^4 + C$$

$$\boxed{\int x^n\,dx \quad \frac{1}{n+1}x^{n+1}}$$

9) $3 \int \frac{1}{3} (x^3 - 3)^4 \cdot 9x^2 \, dx$

$3 \cdot \int 3x^2 (x^3 - 3)^4 \, dx$

$3 \cdot \frac{1}{5} (x^3 - 3)^5 + C$

$\frac{3}{5} (x^3 - 3)^5 + C$

10) $3 \int \frac{1}{3} 48x^3 (4x^4 - 3)^3 \, dx$

$16x^3$

$3 \int 16x^3 (4x^4 - 3)^3 \, dx$

$3 \cdot \frac{1}{4} (4x^4 - 3)^4 + C$

$\frac{3}{4} (4x^4 - 3)^4 + C$

ex/ $\frac{1}{3} \int 3 \cdot 2x^5 (x^6 + 1)^3 \, dx$

$6x^5$

$\frac{1}{3} \int 6x^5 (x^6 + 1)^3 \, dx$

$\frac{1}{3} \cdot \frac{1}{4} (x^6 + 1)^4$

$\frac{1}{12} (x^6 + 1)^4 + C$

$\int x^3 \, dx$

$\frac{1}{4} x^4 + C$

$\int x^n \, dx$

$\frac{1}{n+1} x^{n+1}$

Rational Functions:
Review: derivative of ln(x) = 1/x
So $\int \frac{1}{x}dx = \ln(x) + C$
But since ln cannot take in negative values, and 1/x can take negative values, we use an absolute value for the natural log part. So really it is:

$$\int \frac{1}{x}dx = \ln|x| + C$$

1) $\int \frac{15x^4}{x^5-2}dx$

$3\int \frac{1}{3}15x^4 \frac{1}{x^5-2}dx$

$5x^4$

$3\int 5x^4 \frac{1}{x^5-2}dx$

ex $\frac{3}{2} \Rightarrow 3\cdot\frac{1}{2}$

$(\approx)^2$

$(\quad)^2$

$\frac{d}{dx}3x^2$

$3\frac{d}{dx}x^2$

Final answer: $3\ln\left|x^5-2\right| + C$

2) $\int -\frac{6x^2}{x^3-5}dx$

$-1\int \frac{6x^2}{x^3-5}dx$

$-1\int 6x^2\frac{1}{x^3-5}dx$

ex $\frac{d}{dx}3x^2$

$3\frac{d}{dx}x^2$

$3\cdot 2x = \boxed{6x}$

Final answer: $-\frac{1}{2}\ln|x^3-5| + C$