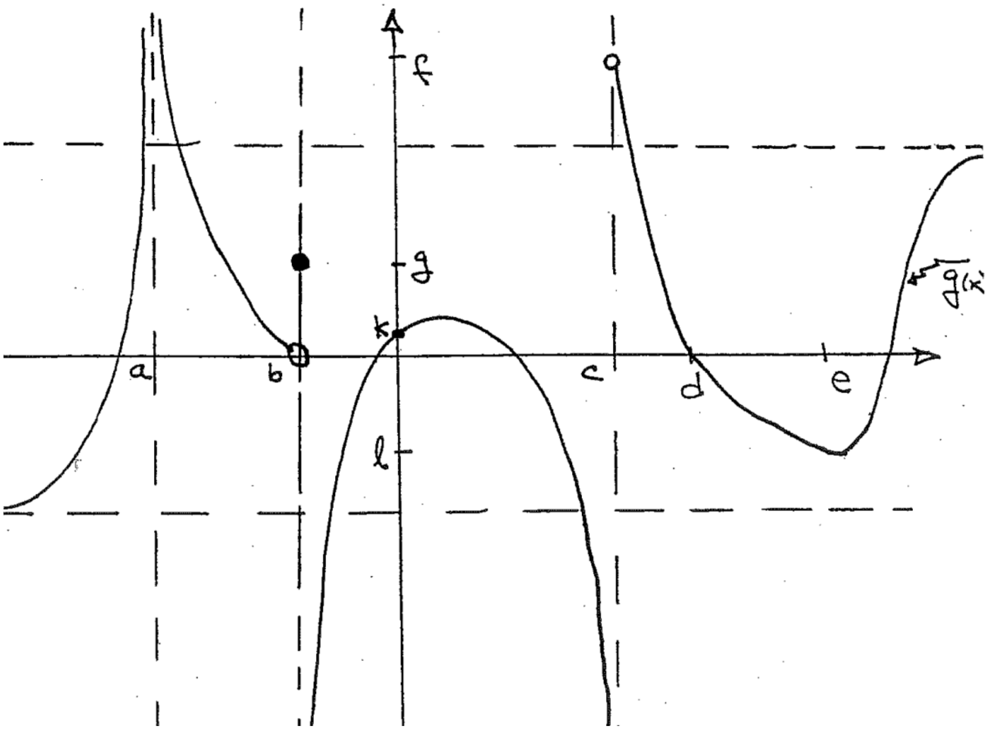
Evaluate limits in three ways: numerically, analytically, and graphically.

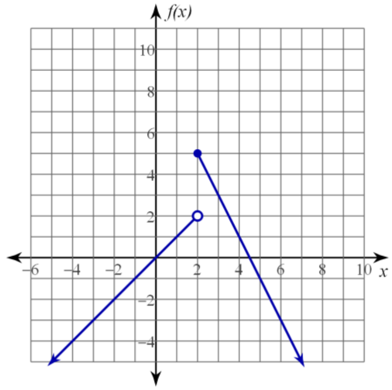
1. Given a generic graph, be able to find the limits (one-sided, two-sided, and infinite).

| | |
|---|---|
| 1. $\lim_{x \to \infty} g(x) =$ | 2. $\lim_{x \to -\infty} g(x) =$ |
| 3. $\lim_{x \to a^+} g(x) =$ | 4. $\lim_{x \to a^-} g(x) =$ |
| 5. $\lim_{x \to a} g(x) =$ | 6. $\lim_{x \to 0} g(x) =$ |
| 7. $\lim_{x \to b^+} g(x) =$ | 8. $\lim_{x \to b^-} g(x) =$ |
| 9. $\lim_{x \to b} g(x) =$ | 10. $\lim_{x \to c} g(x) =$ |
| 11. $\lim_{x \to d} g(x) =$ | 12. $\lim_{x \to e} g(x) =$ |
| 13. g(e) = | 14. g(0) = |
| 15. g(b) = | |




2. Given the graph of a specific function, be able to calculate its limits

$$\lim_{x \to 2^-} f(x),\ f(x) = \begin{cases} x, & x < 2 \\ -2x + 9, & x \geq 2 \end{cases}$$



3. Be able to rewrite an absolute value function as a piecewise function and then find its limits.

$$f(t) = \frac{-5|x+3|}{-x-3}$$

4. Numerically estimate the limit using a table of values.

"Use the provided table to numerically estimate $\lim_{x\to -3} \frac{x^2-9}{x+3}$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | | | | -3 | | | |
| f(x) | | | | undef. | | | |

5. Use direct evaluation to find limits; factor or rationalize functions to allow for direct substitution.

$$\lim_{x\to 0} \frac{x^2-4}{x^2-6x+8}$$

$$\lim_{x\to -1} \frac{x^2+3x+2}{x^2+1}$$

$$\lim_{x\to 4} \frac{4x^2-64}{x-4}$$

$$\lim_{x\to 2} \frac{\sqrt{x-1}-1}{x-2}$$

6. Use properties of limits to evaluate.

If $\lim_{x\to\sqrt{2}} f(x) = \frac{1}{3}$ *and* $\lim_{x\to\sqrt{2}} g(x) = -\frac{1}{2}$ find $\lim_{x\to\sqrt{2}}[f(x) * g(x)]^2$

7. Evaluate limits at and resulting in, infinity.

$$\lim_{x\to 3} \frac{3}{x^2-6x+9}$$

$$\lim_{x\to -5^-} -\frac{1}{x+5}$$

$$\lim_{x\to\infty} \frac{5x+2x^2}{-7-9-3x^2}$$

$$\lim_{x\to 0} 3x+2+\frac{1}{x^2}$$

8. Find the vertical and horizontal asymptotes of given functions and justify your answers.
   Definition of VA: A function has a vertical asymptote at a finite value *c* if lim as x->c from either direction yields positive or negative ∞. There may be many such values of *c*, resulting in many V.As.

   Definition of HA: A function has a horizontal asymptote if the limit as x->∞ of the function yields a finite value *b* , or if the limit as x->−∞ of the function yields a finite value *c*. A function may have zero, one, or two H.A.s, and both may occur at the same value (meaning b=c).

Find the horizontal asymptotes. Justify your answer analytically.

$$f(x) = \begin{cases} \dfrac{3x^2-2x+5}{-7x^2-9x+9} & if\ x < -2 \\ e^x & if\ x = 2 \\ \dfrac{9-2x}{5-4x+3x^2} & if\ x > 2 \end{cases}$$

Find all the vertical asymptotes. Justify your answer analytically.

$$g(x) = \frac{2x-2}{(x-1)(x^2+4x-45)}$$

9. Given a function, determine if its points of discontinuity are jump, removable, or infinite.
   a. Jump: lim from both sides are finite, but equal different values (metaphor: roads get to river, but don't meet)
   b. Removable: lim exists (i.e. both roads meet) but the$\lim_{x\to c} f(x) \neq$ f(c) (either because the function is undefined at c (no bridge at all) or f(c) is defined as a value unequal to the limit (bridge in the wrong place)
   c. Infinite: a vertical asymptote; either side of the limit approaches pos or neg infinity. (one or both roads never reach the river).

Find the values of x for which f(x) is discontinuous and label these as jump, removable, or infinite.

$$f(x) = \frac{x-2}{x^2-4}$$

10. Find the values of c to make a function continuous over the entire number line.
   a. Continuity: 3 parts! (road = road = bridge)
   b. Demonstrate that: $\lim_{x \to c-} f(x) = \lim_{x \to c+} f(x) = f(c)$

Find the value of c so that f(x) is continuous over all real numbers.

$$f(x) = \begin{cases} x^2, & x \leq 3 \\ \frac{c}{x}, & x > 3 \end{cases}$$