

Continuity and Piecewise Functions

Name: _____

Now that you're in calculus class, you're always seeing the math wherever you go. Just the other day, you were at Publix, where they were trying to sell some extra avocados, so they had a promotion going on. "Buy 4 avocados for \$1.98 a piece, and get each one after that for half off. Limit: 10 avocados per customer."

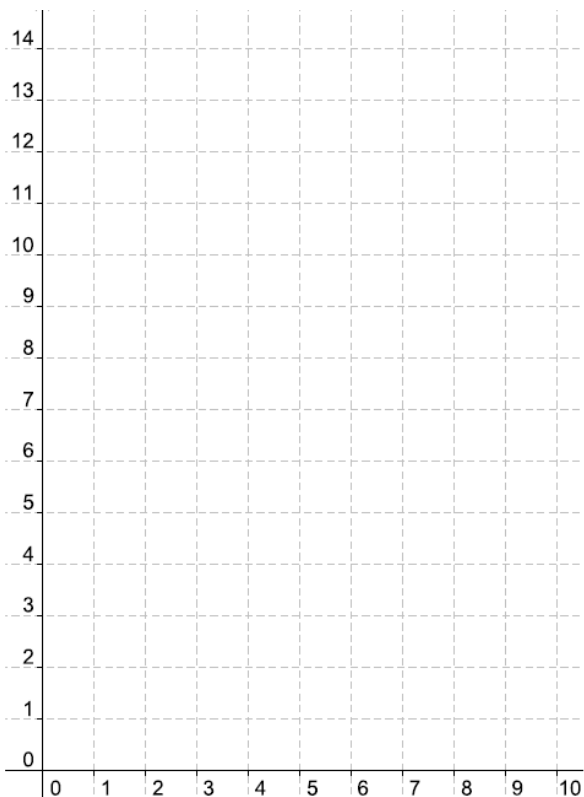
"Wait a minute!" you shouted. "The price of avocados is a piecewise function." You are very clever. You scare the other shoppers at Publix a little bit, but you are very clever.

Of course, right away you wanted to write a function definition for your piecewise function, but you felt a little bit stuck, so you decided to start with a table.

Number of Avocados	1	2	3	4	5	6	7	8	9	10
Total Cost										

Before you go any further, be aware that Publix employees will laugh at you when you ask to buy a part of an avocado. Describe the domain of your function:

There's a grocery store down the street (Real Groceries) that will cut the avocados and sell you any fraction (or irrational piece) of an avocado. Otherwise, the price is identical at both stores. Describe the domain of the function at Real Groceries:



Use the grid to graph the total cost of avocados vs. number of avocados at Real Groceries.

Is the cost of avocados at Publix continuous?

Why or why not?

Is the cost of avocados at Real Groceries continuous?

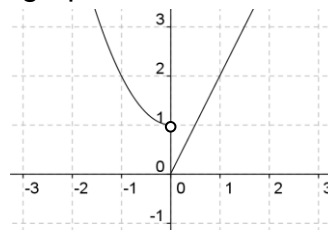
Why or why not?

Now, you're finally ready! Write the piecewise function for the cost of avocados at Real Groceries:

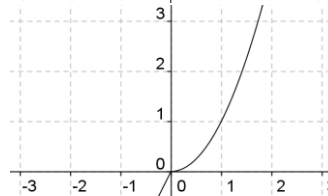
$$C(a) = \left\{ \right.$$

Write the name of the piecewise function next to its graph:

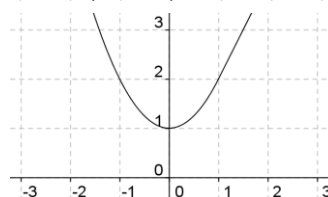
$$f(x) = \begin{cases} 2x & \text{for } x \leq 0 \\ x^2 & \text{for } x > 0 \end{cases}$$



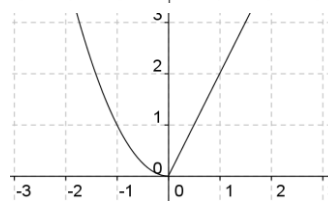
$$g(x) = \begin{cases} 2x & \text{for } x \geq 0 \\ x^2 & \text{for } x < 0 \end{cases}$$



$$h(x) = \begin{cases} 2x & \text{for } x \geq 0 \\ x^2 + 1 & \text{for } x < 0 \end{cases}$$



$$k(x) = \begin{cases} 2x & \text{for } x \geq 1 \\ x^2 + 1 & \text{for } x < 1 \end{cases}$$



Which of the functions is *not* continuous?

Explain how you can tell from the graph whether a piecewise function is continuous.

Explain how you can tell from the function definition whether a piecewise function is continuous.

OK, now. You've bought avocados at Real Grocery store. You've matched graphs to piecewise functions. It's time for the biggest challenge of all. Pick your favorite number from the set of numbers $\{1, 2, 3, 4, 5\}$. Write it down.

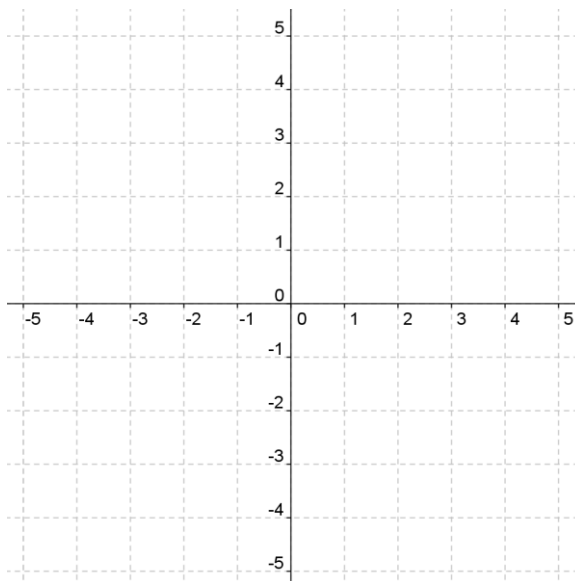
For the next problem k stands for your favorite number (written above), and $g(x)$ is the piecewise function defined below.

Graph $g(x)$.

$$g(x) = \begin{cases} 0.5x & \text{for } x \geq 2 \\ x^2 - k & \text{for } x < 2 \end{cases}$$

Is $g(x)$ continuous?

For what value of k would $g(x)$ be continuous?



For each of the following problems find the value of the constant k that would make $h(x)$ continuous.

$$h(x) = \begin{cases} \sin x & \text{for } x \geq 0 \\ \cos x + k & \text{for } x < 0 \end{cases} \quad k = \underline{\hspace{2cm}}$$

$$h(x) = \begin{cases} x^2 + k & \text{for } x \leq 2 \\ x^3 & \text{for } x > 2 \end{cases} \quad k = \underline{\hspace{2cm}}$$

$$h(x) = \begin{cases} x^3 - 2x - 5 & \text{for } x < 2 \\ x^2 + x + k & \text{for } x \geq 2 \end{cases} \quad k = \underline{\hspace{2cm}}$$

$$h(x) = \begin{cases} 2x^2 & \text{for } x \geq -1 \\ kx & \text{for } x < -1 \end{cases} \quad k = \underline{\hspace{2cm}}$$

Write a description in words of how you approached these problems.